Chapter Four

Lenses

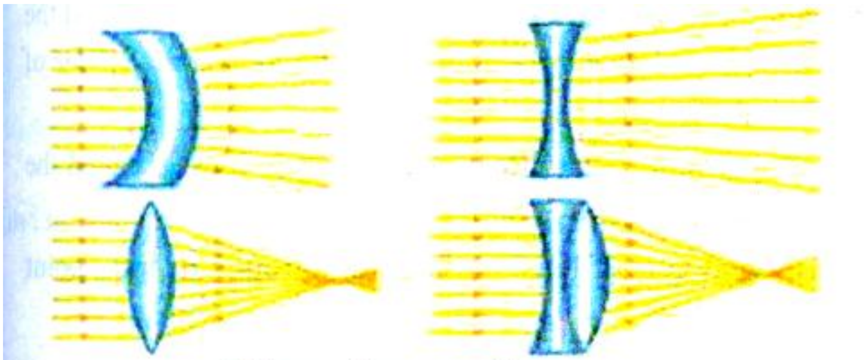
4.1 INTRODUCTION

A Lens is an image-forming device. It forms an image by refraction of light at its two bounding surfaces. ***In general, a lens is made of glass and is bounded by two regular curved surfaces; or by one spherical surface and a plane surface.***

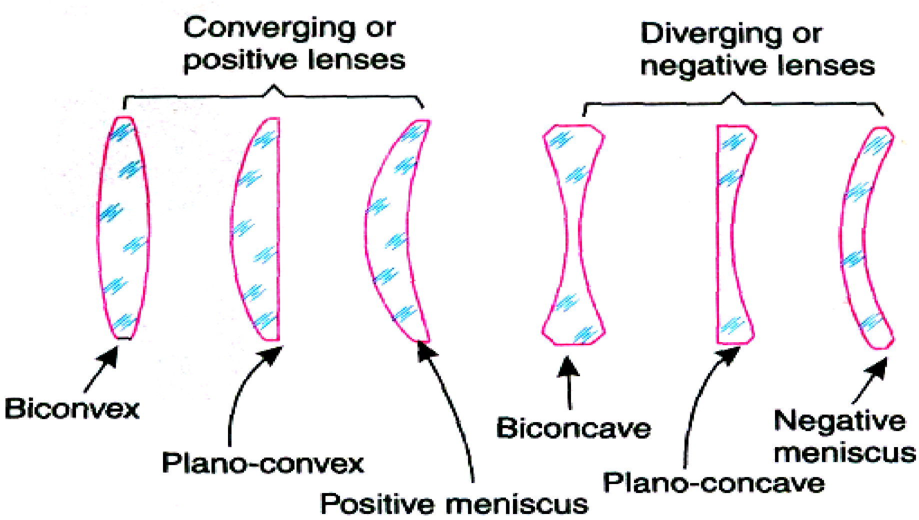
LENSES

Lenses are mainly of two types- *convex lens* and *concave* lens.

	convex lens	concave lens
1	thicker at the center than at the edges	thinner at the center than at the edges
2	convex lens is called a *converging lens*	concave lens is called a *diverging lens*



Different types of lenses.



4.2 TERMINOLOGY

We first acquaint with the terminology and the sign convention associated with lenses.

• A lens has two curved surfaces, each surface having a **curvature.**

• The length of the radius of curvature of surface is called the **radius of curvature, R.**

• The reciprocal of the length of the radius of curvature is known as the **curvature C**(C= 1/R). A lens has two centers of curvature and two radii of curvature, one for each refracting surface.

• The line joining the centers of curvature of the two curved surfaces is called the **principal axis.**

• The points where the principal axis intersects the two refracting surfaces are called the **front vertex** and the **back vertex.**

• The point **F** to which a set of rays parallel to the principal axis is caused to converge (in case of convex lens) or appear to diverge (in case of concave lens) is the **principal focus.**

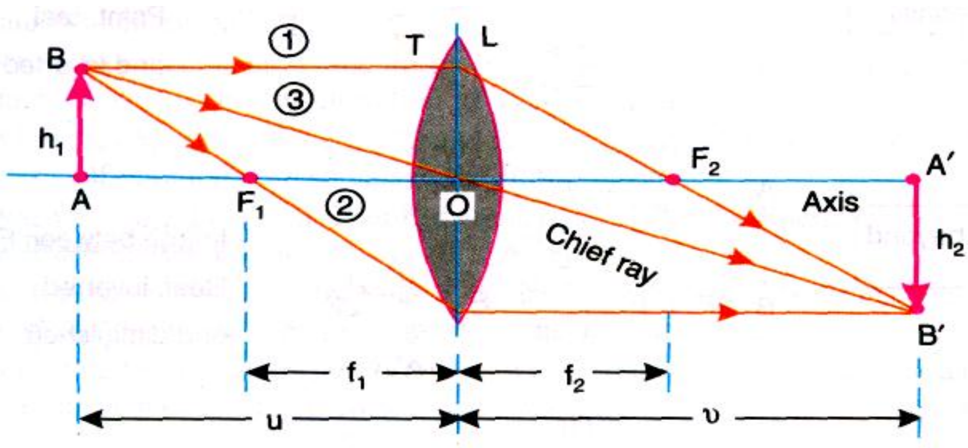
• For every lens, there is a point on the principal axis for which the rays passing through it are not deviated by the lens. Such a point is called the **optical centre.**

• The distance between the focal point **F** and the optical center of the lens is called the **focal length** of the lens.

• The plane perpendicular to the principal axis of lens and passing through its focal point is known as the **focal plane.**

4.3 IMAGE TRACING

We may use graphical ray tracing to determine the position of the image formed by a lens. To find the image, we take the help of characteristic rays shown in Fig.

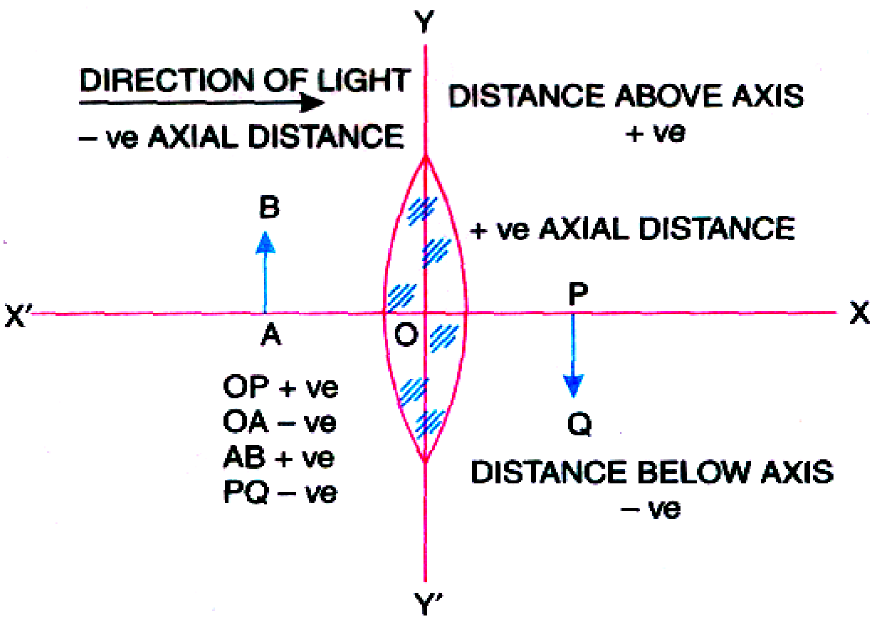


1. One is the ray parallel to the principal axis, which after refraction, passes through focal point $\mathbf{F_2}$.

2. Second ray is the ray that passes through the first focal point **F**, of the lens; after refraction, it travels parallel to the principal axis.

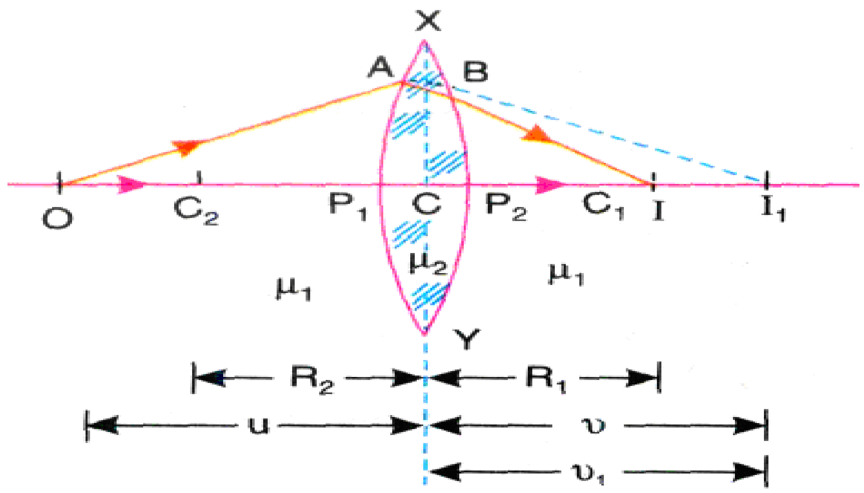
3. The third ray, usually called ***chief ray*** goes through the optical centre of the lens and emerges without deviation. Using any two of the three characteristic rays, we can readily determine the image of any object-point or of any extended object.

4.6 SIGN CONVENTION



4.7 THIN LENS

Lenses are broadly classified into ***thin*** and ***thick*** lenses. **A *lens is said to be* thin *if the thickness of the lens can be neglected when compared to the lengths of the radii of curvature of its two refracting surfaces, and to the distances of the objects and images from it.*** No lens is actually a thin lens. Yet many simple lenses commonly used can be treated as equivalent to a thin lens.



$$\frac{1}{v}-\frac{1}{u}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]\text{...........}(4.1)$$

4.8 LENS MAKER'S EQUATION

If the object is at infinity, the image will form at the principle focus of the lens. When

$$u=\infty, \frac{1}{u}=0 \text{ and } v=f$$

Equation (4.1) become

$$\frac{1}{f}-\frac{1}{\infty}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

$$\frac{1}{f}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]\text{.......................}(4.2)$$

Equation (4.2) is known as the **<u>lens makers' formula.</u>**

Now by comparing equation (4.1) and (4.2) we see that:

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\text{............}(4.3)$$

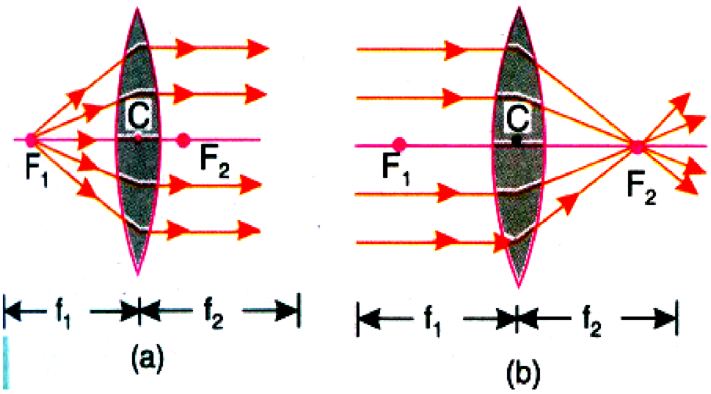
The above equation is known as the **<u>Gauss formula for a lens.</u>**

Using sign convention we get: $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

4.8.1. POSITIONS OF THE PRINCIPAL FOCI

Each individual surface of the lens has its own focal points and planes and the lens as a whole has its own ***pair*** of focal points and focal planes. The focal points and focal planes of the lens are known **as principal focal points** and **principal focal planes**.

(i) If a point object is placed on the principal axis such that the rays refracted by the lens are parallel to the axis, then the position of the point object is called the **first principal focus** $\mathbf{F_1}$ (see Fig. (a)) of the lens.



The distance at the first principal focus from the optical center **C** of the lens is called the **first principal focal length** f_1. We can find f_1 as follows the plane perpendicular to the axis and passing through the first focal point is known as the **first principal focal plane.**

Using $u = f_1$, and $\upsilon = \infty$ into equ. (4.1), we get

$$\frac{1}{\infty}-\left(-\frac{1}{f_1}\right)=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

or

$$\frac{1}{f_1}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right] \quad \text{.........................(4.4)}$$

(ii) If the object is situated at infinity, the position of the image on the axis is known as the **second principal focus F_2** (see Fig. (b)). the distance of the second principal focus from the optical center **C** is called the **second principal focal length, f_2.**

Using $u = \infty$ and $\upsilon = f_2$ into equ. (4.1) , we get

$$\frac{1}{f_2} - \frac{1}{\infty} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{f_2} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \text{..........}(4.5)$$

The plane perpendicular to the axis and passing through the second focal point is known as the **second principal focal plane.**

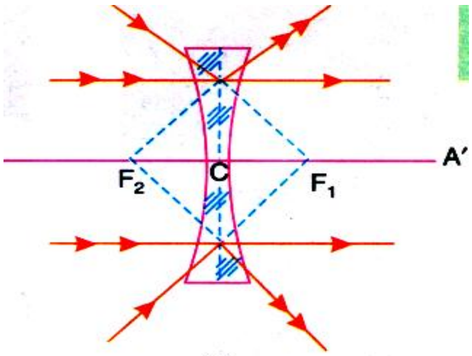
It follows from equ.(4.4) and (4.5) that

$$f_1 = f_2 \text{. ....}(4.6)$$

Thus, ***every thin lens in air has two focal points (F_1 and F_2), one on each side of the lens and equidistant from the centre.***

It will be seen that the second focal, length (***f_2***) of a converging lens is positive and the first ***(f_1)*** negative, while for a diverging lens the reverse is true (see Fig.4.8c).

The two focal lengths of thin lens in air are numerically equal.

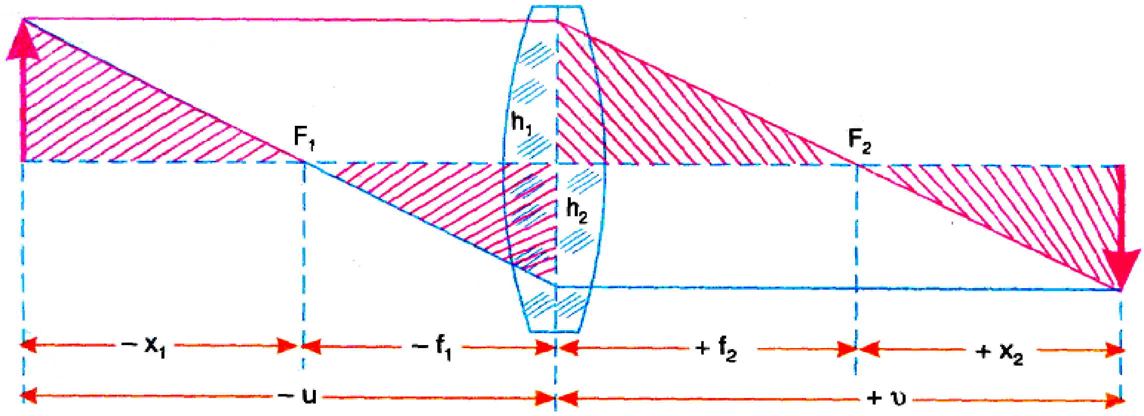


(c)

4.9 NEWTON'S LENS EQUATION

Let $\mathbf{h_1}$, be the height of the object and $\mathbf{h_2}$ be the image height. From the similar triangles to the left of the lens we find that:

$$\frac{h_1}{x_1} = \frac{h_2}{f_1} \Rightarrow \frac{h_1}{h_2} = \frac{x_1}{f_1}$$



To the right of the lens we have:

$$\frac{h_1}{f_2} = \frac{h_2}{x_2} \Rightarrow \frac{h_1}{h_2} = \frac{f_2}{x_2}$$

Combining both equations by eliminating h_1/h_2

$$\frac{x_1}{f_1} = \frac{f_2}{x_2}$$

$$x_1 x_2 = f_1 f_2 \text{....................}(4.7)$$

When a medium is the same on both sides of the lens the equation reduces to:

$$x_1 x_2 = f^2 \text{....................}(4.8)$$

This is known as **Newton's lens equation**.

4.10 MAGNIFICATION

Magnification is defined as: $$m = \frac{size\ of\ image}{size\ of\ object}$$

We distinguish three types of magnification, namely ***lateral magnification, longitudinal magnification and angular magnification***.

1. LATERAL MAGNIFICATION

Lateral or **transverse magnification** of a lens is defined as the ratio of the length of the image to the length of the object size.

$$m = \frac{h_2}{h_1} = \frac{v}{u} \text{...............}(4.9)$$

According to sign convention, the distances above the principal axis of the lens are taken positive and those below the axis are negative. Hence, the lateral magnification is positive for an erect image and negative for an inverted image.

The lateral magnification corresponding to Newton's formula may be written as:

$$m = \frac{h_2}{h_1} = \frac{f_1}{x_1} = \frac{x_2}{f_2} \text{...............}(4.10)$$

2. LONGITUDINAL MAGNIFICATION

The longitudinal magnification is defined as the ratio of an infinitesimal axial length in the region of the image to the corresponding length in the region of the object.

$$m_L = \frac{dx_i}{dx_\circ} \text{................}(4.11)$$

Differentiating equation (4.8) we get:

$$m_L = -\frac{f^2}{x_\circ^2} = -m^2 \text{.............}(4.12)$$

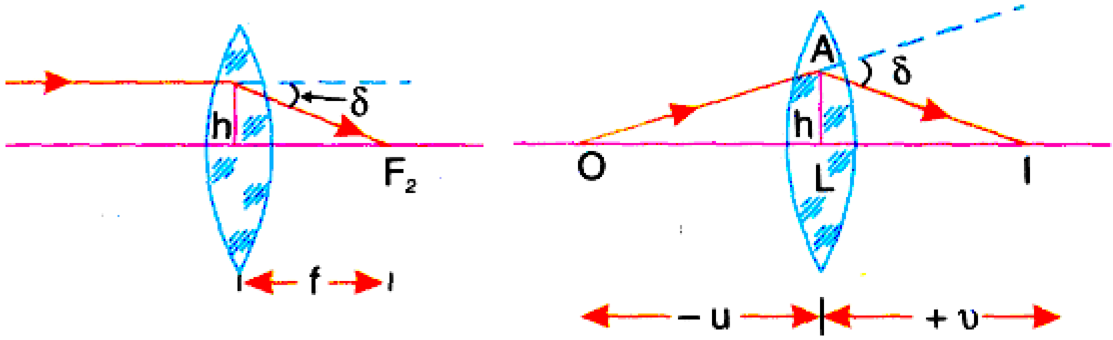
3. ANGULAR MAGNIFICATION

Angular magnification is defined as the ratio of slopes of emergent ray and conjugate incident ray with the principal axis.

$$\gamma = \frac{\tan\theta_2}{\tan\theta_1} \text{................}(4.13)$$

4.11 DEVIATION BY A THIN LENS

A lens may be considered to be made up of a large number of prisms placed one above the other. It is necessary to find the deviation produced by a particular section of the lens. Let a ray of monochromatic light parallel to the principal axis be incident on a thin lens, after refraction it will pass through the 'second focus, $\mathbf{F_2}$ (see Fig. (a)).



(a) (b)

$$\tan\delta = \frac{h}{f}$$

In the paraxial region δ begin small then $\tan\delta = \delta$

$$\delta = \frac{h}{f}$$

The deviation suffered corresponding to the ray OA incident at A is given by (see Fig.(b)):

$$\delta = \angle AOL + \angle AIL$$

$$\delta = \frac{h}{-u} + \frac{h}{+v} = h\left[\frac{1}{v} - \frac{1}{u}\right] = +h\left[\frac{1}{f}\right]$$

$$\delta = \frac{h}{f} \text{..............................} (4.14)$$

This shows that the deviation produced by a lens is *independent* of the position of the object.

4.12 Power

The power of a lens is the measure of its ability to produce convergence of a parallel beam of light. The unit in which the power of a lens is measured is called a **diopter** (D).

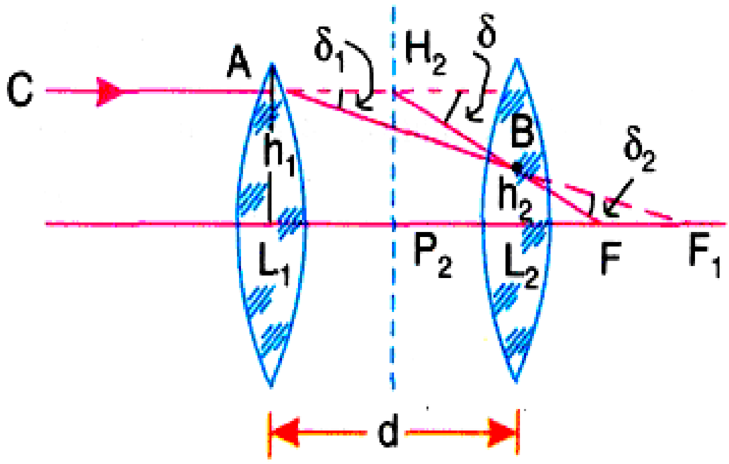
$$mathematically \quad power = \frac{1}{Focal\ length\ in\ meter}$$

The power of a pair of lenses of focal lengths f_1 and f_2 placed in contact is equal to:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$P = P_1 + P_2 \ldots\ldots\ldots\ldots\ldots\ldots(4.15)$$

4.13 EQUIVALENT FOCAL LENGTH OF TWO THIN LENSES



When two thin lenses are arranged coaxially, the image formed by the first lens system becomes the object for a second lens system and the two systems act as a single optical system forming the final image from the original object.

We find that two lenses, separated by a finite distance, can be replaced by a single thin lens called an **equivalent lens.** The equivalent lens, when placed at a suitable fixed point, will produce an image of the same size as that produced by the combination of the two lenses. The focal length of equivalent lens is called **equivalent focal length.** We now derive an expression for the equivalent focal length of the combination of two lenses.

1. Focal Length of the Equivalent Lens

Deviation produced by the first lens L_1 is $\delta_1 = h_1 / f_1$

Deviation produced by the second lens L_2 is $\delta_2 = h_2 / f_2$

But $$\delta = \delta_1 + \delta_2$$

$$\therefore \quad \frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_2}{f_2}$$

The Δ^{les} AL_1F_1 and BL_2F_1 are similar.

$$\therefore \quad \frac{AL_1}{L_1F_1} = \frac{BL_2}{L_2F_1}$$

or $$\frac{h_1}{f_1} = \frac{h_2}{f_1 - d}$$

or $$h_2 = \frac{h_1(f_1 - d)}{f_1}$$

Using equation (4.29) into equ.(4.27), we get

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1(f_1 - d)}{f_1 f_2}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{f_1 - d}{f_1 f_2}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Therefore, the equivalent focal length is given by

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} \qquad \text{.............................(4.16)}$$

or $$f = \frac{-f_1 f_2}{\Delta}$$

where $\Delta = d - (f_1 + f_2)$ and is known as the **optical interval** between the two lenses.

Note:

1. Equ. (4.16) shows that if the distance ***d*** between the two convex lenses exceeds the sum of their focal lengths *(f_1 + f_2)*, the system becomes divergent, because of negative focal length.

2. If the medium between the two convex lenses is other than air, then the equ.(4.16) for equivalent focal length would become:

$$f = \frac{f_1 f_2}{f_1 + f_2 - \frac{d}{\mu}}$$

Where μ is the refractive index of the medium.

2. DISTANCE OF EQUIVALENT LENS FROM L_2

Let us say the plane EP_2 is located at a distance of L_2P_2 from the second lens L_2. Now consider the similar Δ^{les} EP_2F and BL_2F.

$$\frac{P_2F}{L_2F} = \frac{EP_2}{BL_2}$$

From the Fig. 4.15 it is seen that $P_2F = f$, $L_2F = f - L_2P_2$ and $EP_2 = AL_1 = h_1$

$$\frac{h_1}{h_2} = \frac{f}{f - L_2P_2}$$

Comparing equations (4.33) and (4.28), we obtain

$$\frac{f}{f - L_2P_2} = \frac{f_1}{f_1 - d}$$

$$ff_1 - fd = ff_1 + (L_2P_2)f_1$$

or $$L_2P_2 = \frac{fd}{f_1} \quad \text{.............................(4.17)}$$

3. POWER

When two thin lenses of focal length f1 and f2 are placed coaxial and separated by a distance d the equivalent focal length is given by:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$P = P_1 + P_2 - d.P_1P_2 \quad \text{..................(4.20)}$$

$\therefore$ $$\upsilon = 7.5 \text{ cm}$$

Therefore, the ray crosses the axis at a point D which is 5 cm from the point P_2 of the globe.