Chapter Four

Lenses

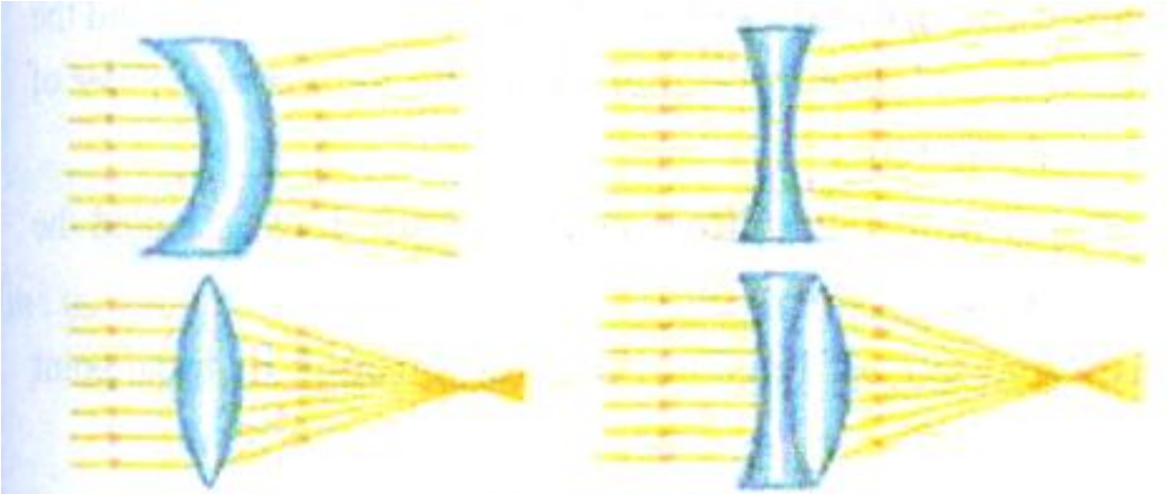
4.1 INTRODUCTION

A Lens is an image-forming device. It forms an image by refraction of light at its two bounding surfaces. ***In general, a lens is made of glass and is bounded by two regular curved surfaces; or by one spherical surface and a plane surface.***

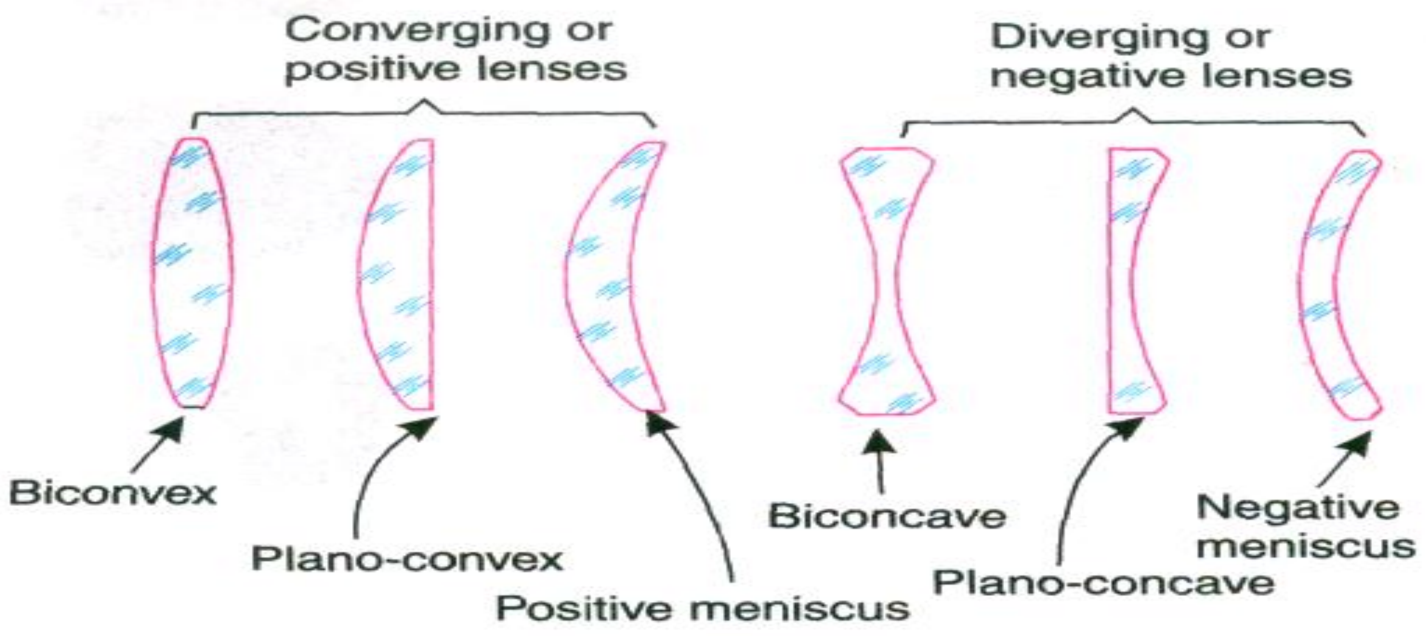
LENSES

Lenses are mainly of two types- *convex lens* and *concave* lens.

	convex lens	**concave lens**
1	thicker at the center than at the edges	thinner at the center than at the edges
2	convex lens is called a *converging lens*	concave lens is called a *diverging lens*



Different types of lenses.



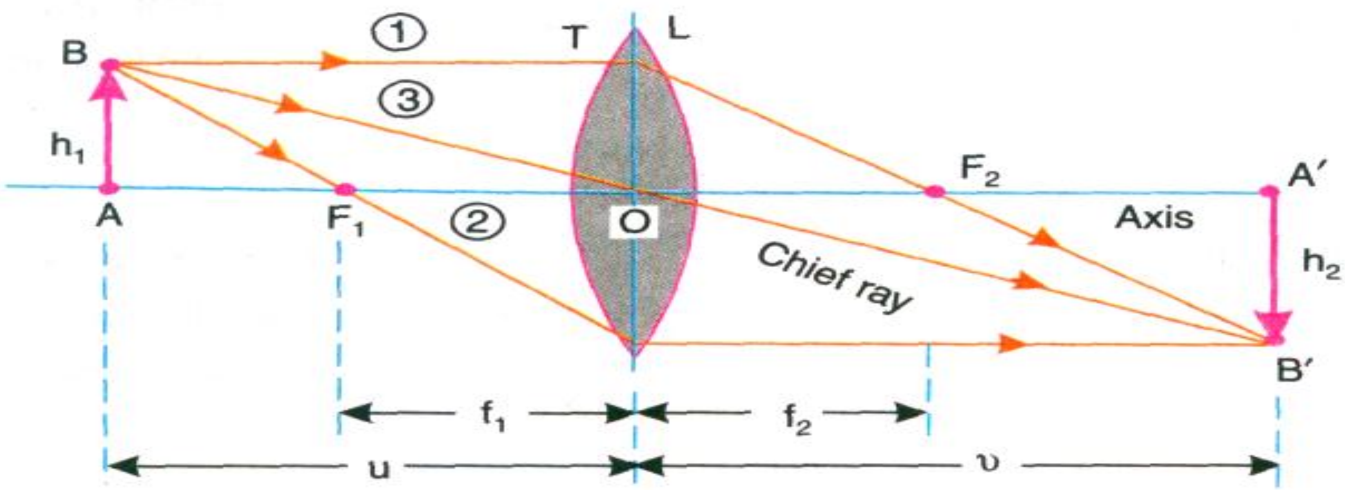
4.2 TERMINOLOGY

We first acquaint with the terminology and the sign convention associated with lenses.

- A lens has two curved surfaces, each surface having a **curvature.**
- The length of the radius of curvature of surface is called the **radius of curvature, R.**
- The reciprocal of the length of the radius of curvature is known as the **curvature C**(C= 1/R). A lens has two centers of curvature and two radii of curvature, one for each refracting surface.
- The line joining the centers of curvature of the two curved surfaces is called the **principal axis.**
- The points where the principal axis intersects the two refracting surfaces are called the **front vertex** and the **back vertex.**
- The point **F** to which a set of rays parallel to the principal axis is caused to converge (in case of convex lens) or appear to diverge (in case of concave lens) is the **principal focus.**
- For every lens, there is a point on the principal axis for which the rays passing through it are not deviated by the lens. Such a point is called the **optical centre.**
- The distance between the focal point **F** and the optical center of the lens is called the **focal length** of the lens.
- The plane perpendicular to the principal axis of lens and passing through its focal point is known as the **focal plane.**

4.3 IMAGE TRACING

We may use graphical ray tracing to determine the position of the image formed by a lens. To find the image, we take the help of characteristic rays shown in Fig.

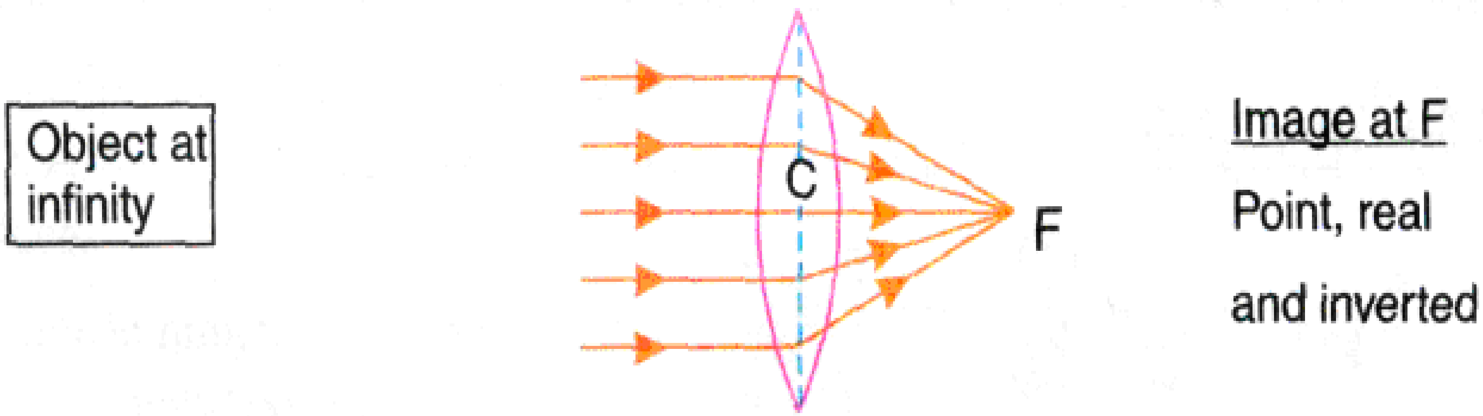


1. One is the ray parallel to the principal axis, which after refraction, passes through focal point F2.
2. Second ray is the ray that passes through the first focal point F, of the lens; after refraction, it travels parallel to the principal axis.
3. The third ray, usually called ***chief ray*** goes through the optical centre of the lens and emerges without deviation. Using any two of the three characteristic rays, we can readily determine the image of any object-point or of any extended object

4.4 LOCATION OF THE IMAGE

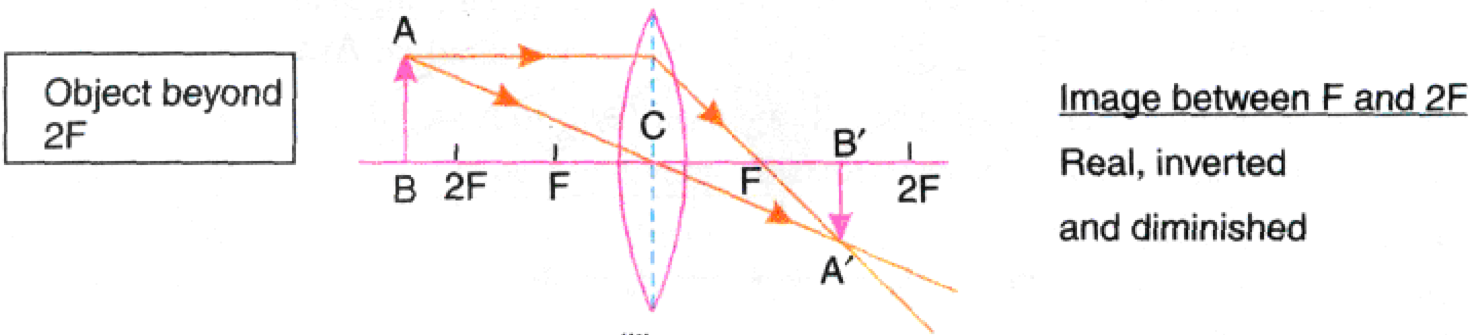
1. When the object at infinity, the image is just to the right of the focal plane. The image is real, inverted, and smaller in size than the object ($m < 1$).

A convex lens produces a real or virtual image depending on the location of the object. A concave lens *always* produces virtual images of real objects.



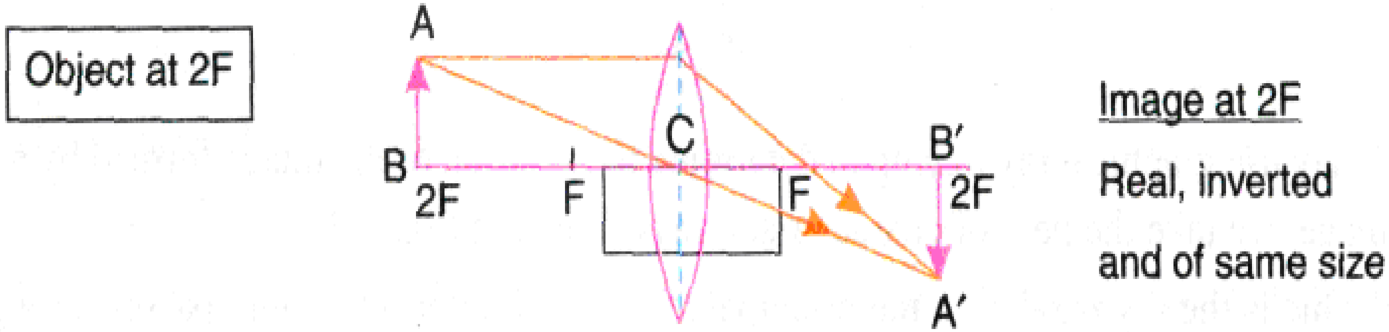
(i)

2. As the object beyond 2F approaches the lens. The *(i)* image is real and inverted. This is the configuration for **cameras and eyeballs**.



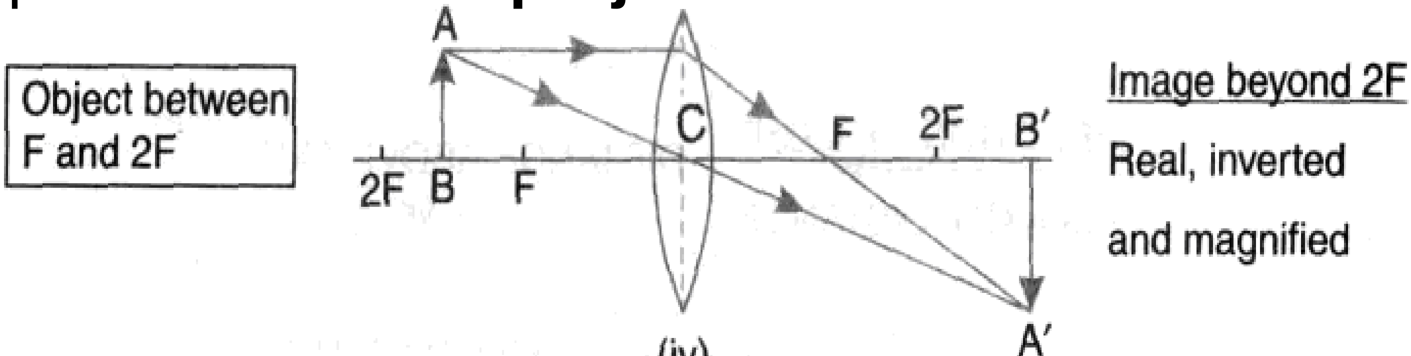
(ii)

3. When the object is at 2F, the image is real, inverted and of the same size as the object (m = 1). This is the configuration of a **photocopier**.



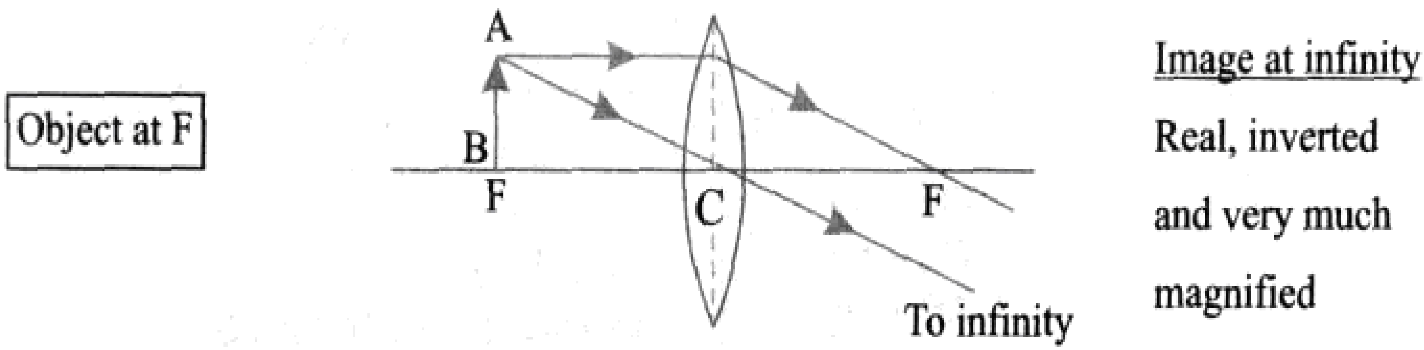
(iii)

4. When the object is in between 2F and F, the image is (i) enlarged (m > 1), real and inverted. This configuration corresponds to the **film projector**.



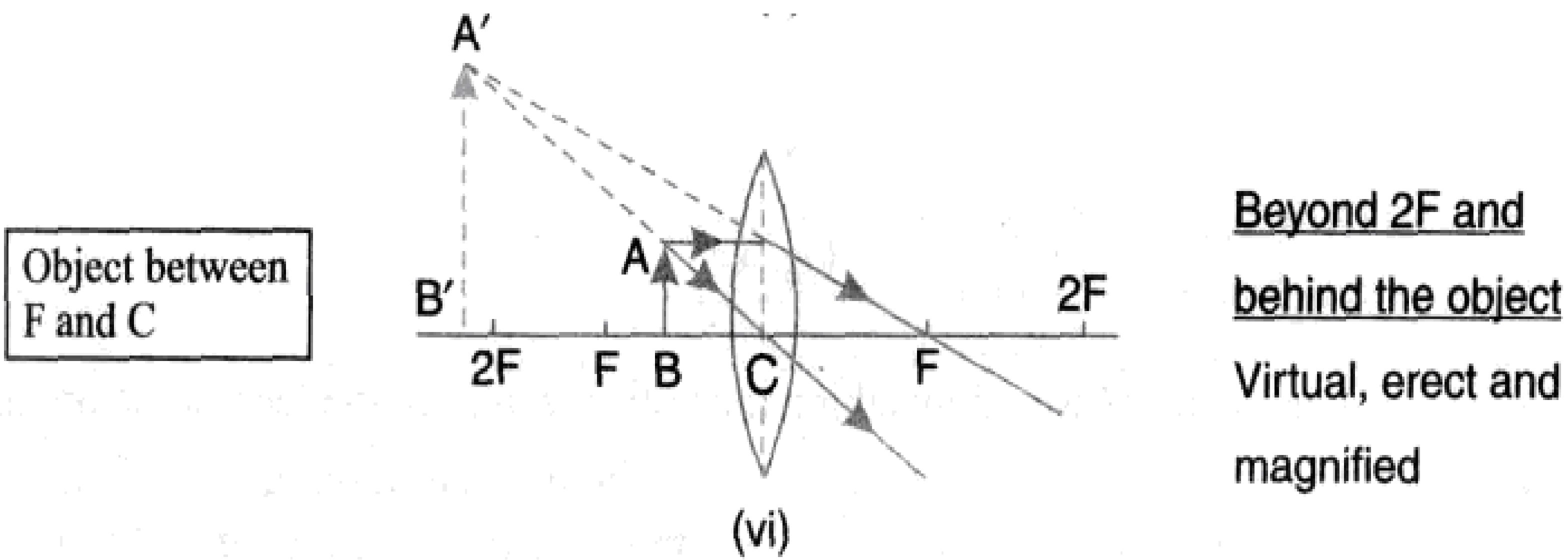
(iv)

5. When the object is precisely at F, there is no (i) image as the emerging rays are parallel in effect the image is at infinity.

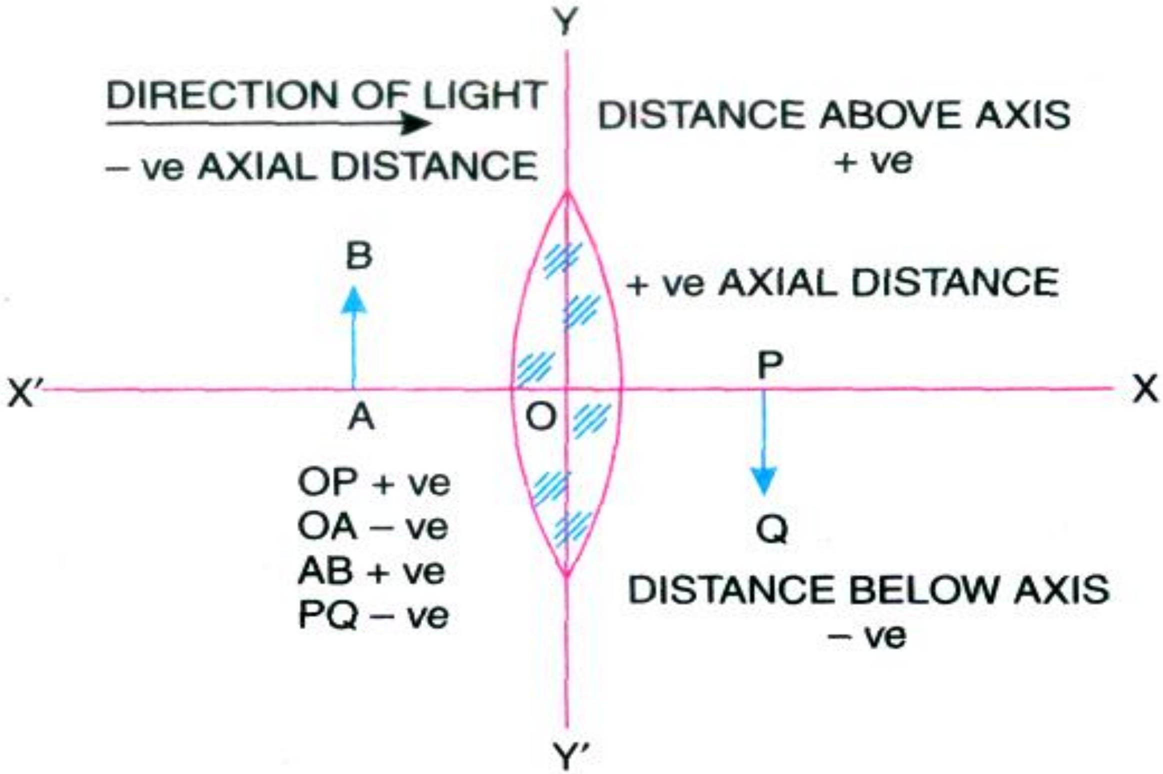


(v)

6. With the object closer in than one F, the image (i) reappears. It is virtual, erect and enlarged (m > 1). This is the configuration of the **magnifying glass**.



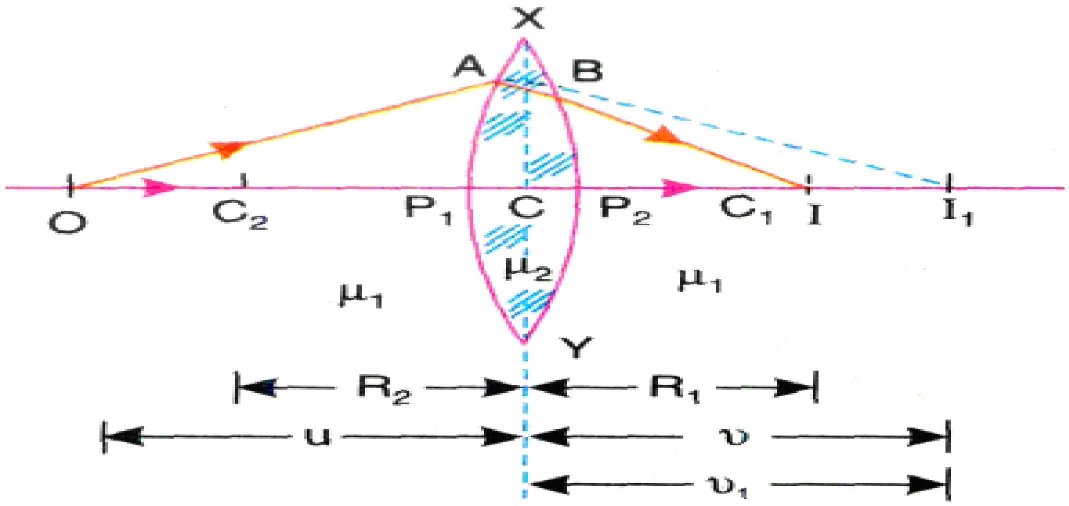
4.6 SIGN CONVENTION



4.7 THIN LENS

Lenses are broadly classified into ***thin*** and ***thick*** lenses. **A *lens is said to be* thin *if the thickness of the lens can be neglected when compared to the lengths of the radii of curvature of its two refracting surfaces, and to the distances of the objects and images from it.***

$$\frac{1}{v}-\frac{1}{u}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]\text{..........}(4.1)$$



4.8 LENS MAKER'S EQUATION

If the object is at infinity, the image will form at the principle focus of the lens. When

$$u = \infty, \frac{1}{u} = 0 \qquad \text{and} \qquad v = f$$

Equation (4.1) become

$$\frac{1}{f} - \frac{1}{\infty} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \text{........................}(4.2)$$

Equation (4.2) is known as the lens makers' formula.

Now by comparing equation (4.1) and (4.2) we see that:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \text{...........}(4.3)$$

The above equation is known as the **Gauss formula for a lens**.

Using sign convention we get:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

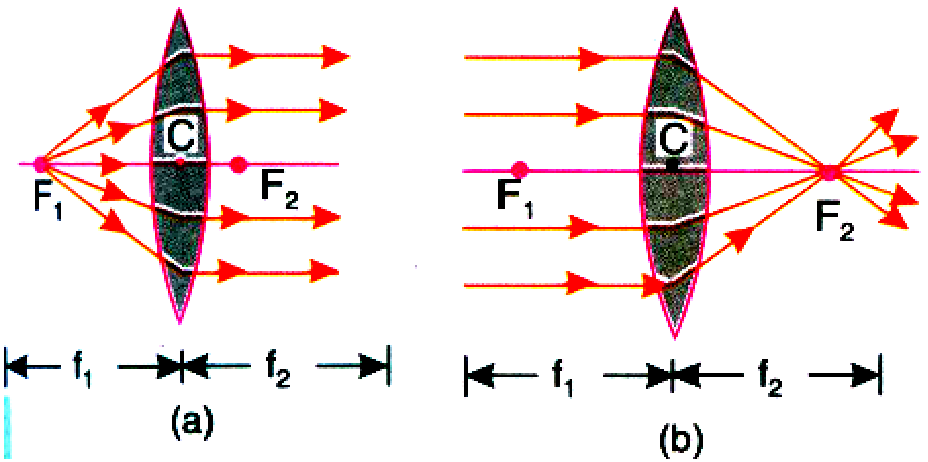
4.8.1. POSITIONS OF THE PRINCIPAL FOCI

Each individual surface of the lens has its own focal points and planes and the lens as a whole has its own *pair* of focal points and focal planes. The focal points and focal planes of the lens are known as principal focal points and principal focal planes.

Using $u = f_1$, and $\upsilon = \infty$ into equ. (4.1), we get

$$\frac{1}{\infty} - \left(-\frac{1}{f_1}\right) = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or $$\frac{1}{f_1} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{.........................(4.4)}$$



Using $u = \infty$ and $\upsilon = f_2$ into equ. (4.1), we get

$$\frac{1}{f_2} - \frac{1}{\infty} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

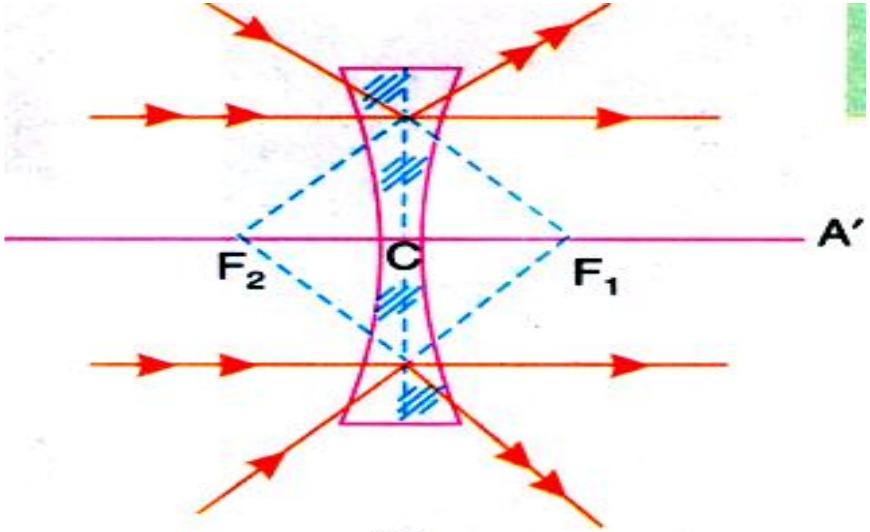
It follows from equ.(4.4) and (4.5) that

$$f_1 = f_{2.} \text{ ……………..(4.6)}$$

Thus, ***every thin lens in air has two focal points (F_1 and F_2), one on each side of the lens and equidistant from the centre.***

It will be seen that the second focal, length (***f2***) of a converging lens is positive and the first ***(f1)*** negative, while for a diverging lens the reverse is true

The two focal lengths of thin lens in air are numerically equal.

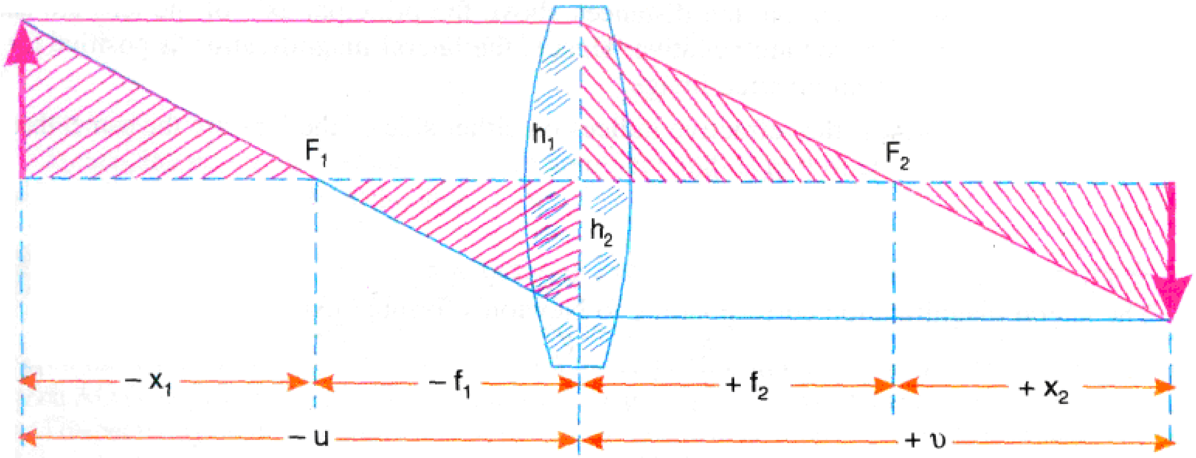


(c)

4.9 NEWTON'S LENS EQUATION

Let h_1, be the height of the object and h_2 be the image height. From the similar triangles to the left of the lens we find that:

$$\frac{h_1}{x_1} = \frac{h_2}{f_1} \Rightarrow \frac{h_1}{h_2} = \frac{x_1}{f_1}$$



To the right of the lens we have:

$$\frac{h_1}{f_2} = \frac{h_2}{x_2} \Rightarrow \frac{h_1}{h_2} = \frac{f_2}{x_2}$$

Combining both equations by eliminating h_1/h_2

$$\frac{x_1}{f_1} = \frac{f_2}{x_2}$$

$$x_1 x_2 = f_1 f_2 \text{.....................} (4.7)$$

When a medium is the same on both sides of the lens the equation reduces to:

$$x_1 x_2 = f^2 \text{.....................} (4.8)$$

This is known as Newton's lens equation.

4.10 MAGNIFICATION

Magnification is defined as:

$$m = \frac{size\ of\ image}{size\ of\ object}$$

Types of magnification:

1- *lateral magnification*

2- longitudinal magnification

3- angular magnification.

4.10.1. LATERAL MAGNIFICATION

Lateral or transverse magnification of a lens is defined as:

$$m = \frac{h_2}{h_1} = \frac{v}{u} \text{...............}(4.9)$$

According to sign convention, the distances above the principal axis of the lens are taken positive and those below the axis are negative. Hence, the lateral magnification is positive for an erect image and negative for an inverted image.

The lateral magnification corresponding to Newton's formula may be written as:

$$m = \frac{h_2}{h_1} = \frac{f_1}{x_1} = \frac{x_2}{f_2} \text{..............}(4.10)$$

4.10.2. LONGITUDINAL MAGNIFICATION

The longitudinal magnification is defined as the ratio of an infinitesimal axial length in the region of the image to the corresponding length in the region of the object.

$$m_L = \frac{dx_i}{dx_\circ} \ldots\ldots\ldots\ldots\ldots (4.11)$$

Differentiating equation (4.8) we get:

$$m_L = -\frac{f^2}{x_\circ^2} = -m^2 \ldots\ldots\ldots\ldots (4.12)$$

4.10.3. ANGULAR MAGNIFICATION

Angular magnification is defined as the ratio of slopes of emergent ray and conjugate incident ray with the principal axis.

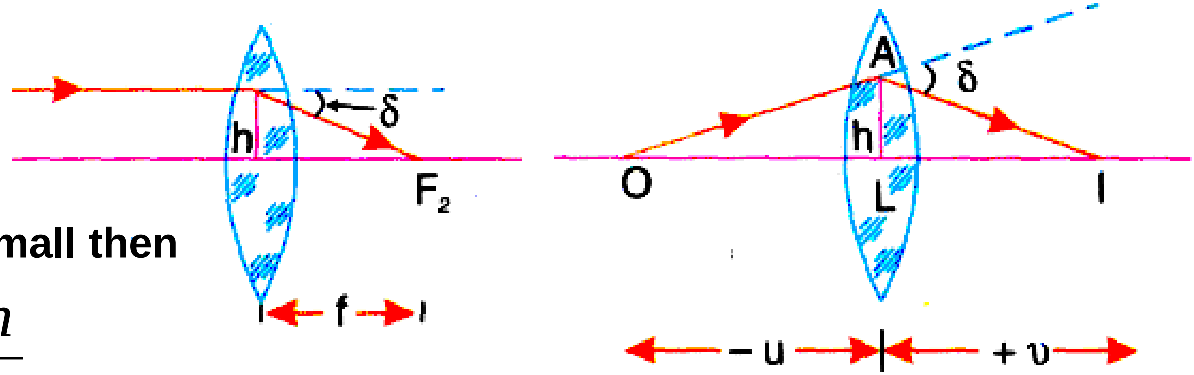
$$\gamma = \frac{\tan\theta_2}{\tan\theta_1} \ldots\ldots\ldots\ldots\ldots\ldots (4.13)$$

4.11 DEVIATION BY A THIN LENS

$$\tan\delta = \frac{h}{f}$$

In the paraxial region δ **begin small then**

$$\tan\delta = \delta \qquad \delta = \frac{h}{f}$$



(a) (b)

The deviation suffered corresponding to the ray OA incident at A is given by (see Fig.(b)):

$$\delta = \angle AOL + \angle AIL$$

$$\delta = \frac{h}{-u} + \frac{h}{+v} = h\left[\frac{1}{v} - \frac{1}{u}\right] = +h\left[\frac{1}{f}\right]$$

$$\delta = \frac{h}{f} \text{..............................} (4.14)$$

This shows that the deviation produced by a lens is *independent* of the position of the object

4.12 Power

The power of a lens is the measure of its ability to produce convergence of a parallel beam of light. The unit in which the power of a lens is measured is called a **diopter** (D).

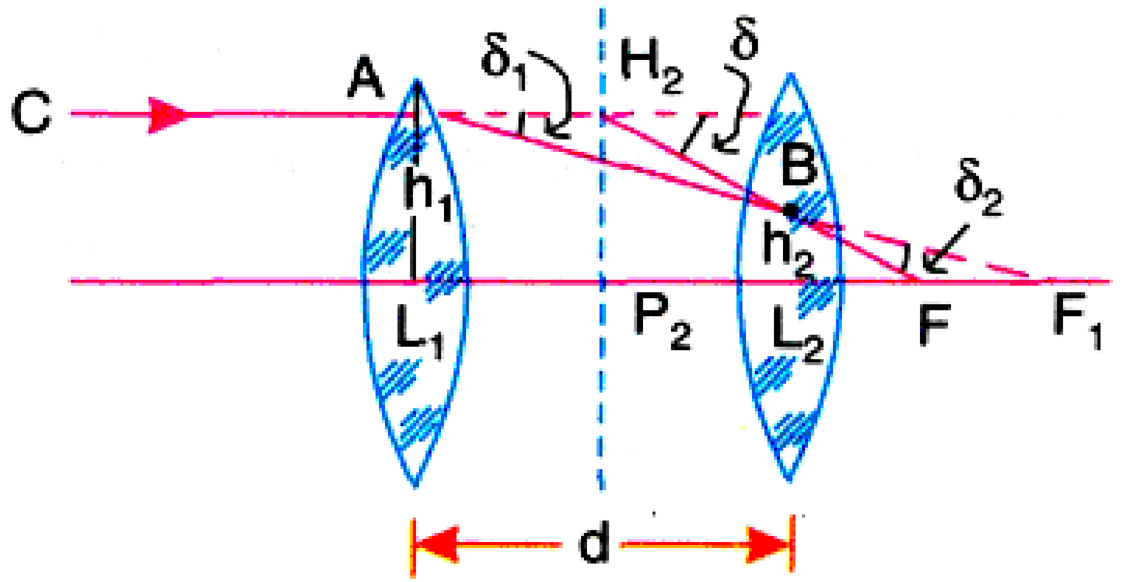
$$mathematically \quad power = \frac{1}{Focal\ length\ in\ meter}$$

The power of a pair of lenses of focal lengths f_1 and f_2 placed in contact is equal to:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$P = P_1 + P_2 \text{....................}(4.15)$$

4.13.1 Focal Length of the Equivalent Lens



4.13.1 Focal Length of the Equivalent Lens

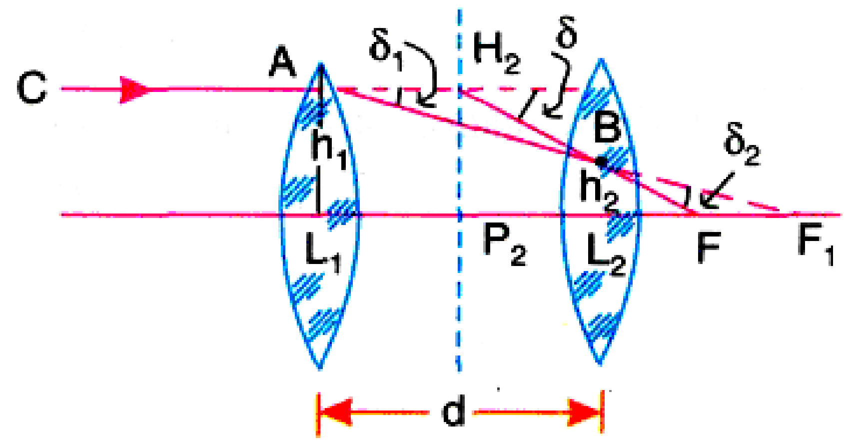
Deviation produced by the first lens L_1 is $\delta_1 = h_1 / f_1$

Deviation produced by the second lens L_2 is $\delta_2 = h_2 / f_2$

But $\delta = \delta_1 + \delta_2$

$\therefore$ $$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_2}{f_2} \quad ....(^*)$$

The Δ^{les} AL_1F_1 and BL_2F_1 are similar.



$\therefore$ $$\frac{AL_1}{L_1F_1} = \frac{BL_2}{L_2F_1}$$

or $$\frac{h_1}{f_1} = \frac{h_2}{f_1 - d}$$

or $$h_2 = \frac{h_1(f_1 - d)}{f_1} \quad ....(^{**})$$

Using equation (*) into equ.(**), we get

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1(f_1 - d)}{f_1 f_2} \Rightarrow \frac{1}{f} = \frac{1}{f_1} + \frac{f_1 - d}{f_1 f_2}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Therefore, the equivalent focal length is given by

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} \quad \text{..............................(4.16)}$$

or $f = \frac{-f_1 f_2}{\Delta}$ where $\Delta = d - (f_1 + f_2)$ and is known as the **optical interval** between the two lenses.

Note:

1. Equ. (4.16) shows that if the distance ***d*** between the two convex lenses exceeds the sum of their focal lengths $(f_1 + f_2)$, the system becomes divergent, because of negative focal length.
2. If the medium between the two convex lenses is other than air, then the equ.(4.16) for equivalent focal length would become:

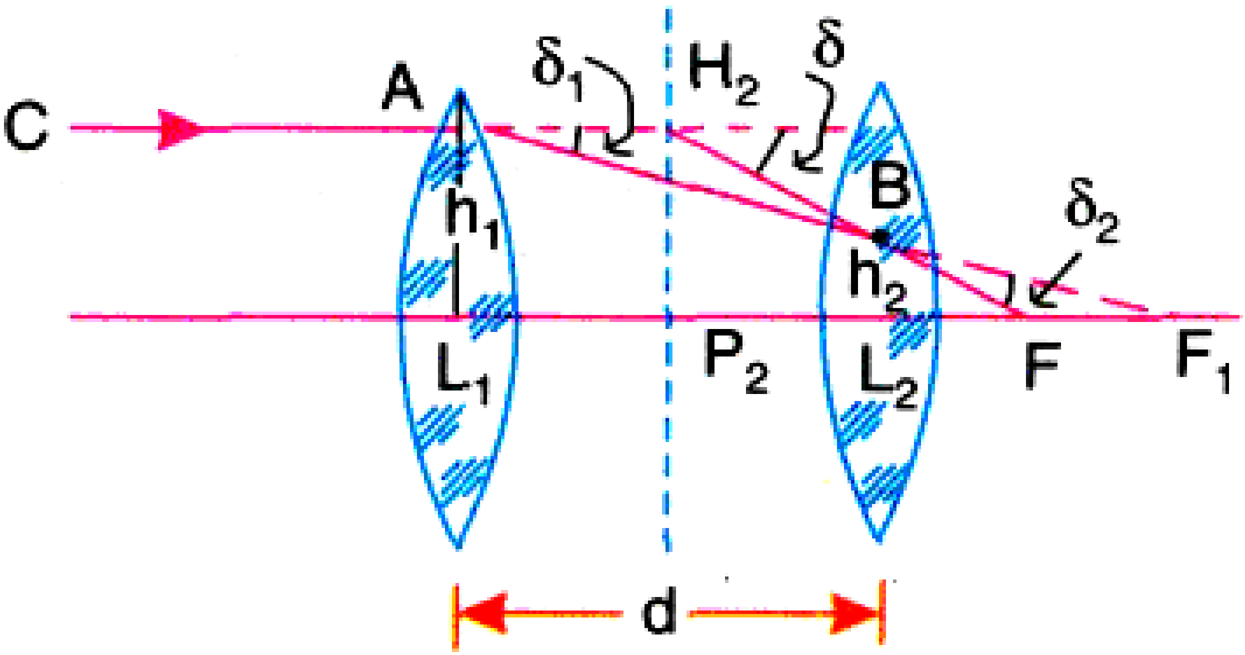
$$f = \frac{f_1 f_2}{f_1 + f_2 - \frac{d}{\mu}}$$

Where μ is the refractive index of the medium.

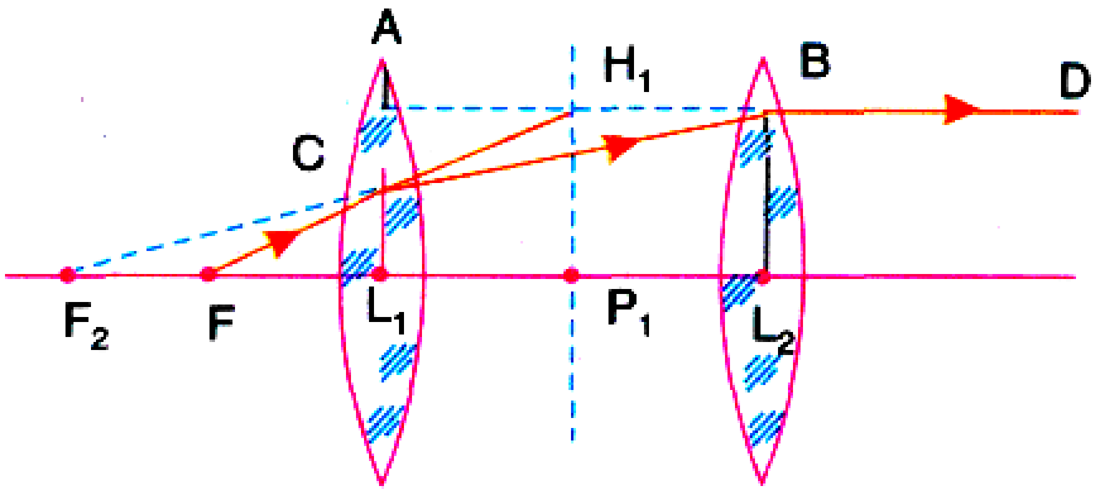
4.13.2 DISTANCE OF EQUIVALENT LENS FROM L_2

Let us say the plane EP_2 is located at a distance of L_2P_2 from the second lens L_2. Now consider the similar

Δ^{les} EP_2F and BL_2F.



4.13.2 DISTANCE OF EQUIVALENT LENS FROM L_1



4.13.3 POWER

When two thin lenses of focal length f1 and f2 are placed coaxial and separated by a distance d the equivalent focal length is given by:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$P = P_1 + P_2 - d.P_1P_2 \quad ....(4.20)$$