Chapter one
Harmonic oscillators

1.1: Introduction:

In every day life we come a cross numerous things that move. These motions are of two types;

(I) The motion in which the body moves about a mean position i.e. a fixed point.

(II) The motion in which the body moves from one place to the other with respect of time.

* The first type of motion is called oscillatory motion (example; oscillating pendulum, vibration of a stretched string , movement of water in a cup , vibration of electron , movement of light in a laser beam etc.)

* A moving train, flying aero plane, moving ball etc. correspond to the second type of motion.

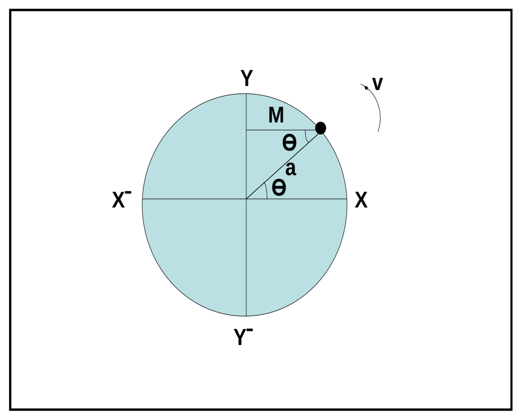
* Sometimes both the types of motion are exhibited in the same phenomenon depending on our point of view.

1.2: Simple harmonic motion:

* Let ***P*** be a particle moving on the circumference of a circle of radius ***a*** with a uniform velocity ***v***

* Let $v = a\omega$
  a is radius
  ω is angular velocity

(The circle along which ***P*** moves is called the circle of reference.)



* (As the particle ***P*** moves round the circle continuously with uniform velocity, the foot of the Perpendicular ***M***, vibrates along the diameter YY'. If the motion of ***P*** is uniform, then the motion of ***M*** is periodic (i.e., it takes the same time to vibrate once between the points y and y'.))

At any instant the distance of ***M*** from the center ***O*** of the circle is called the displacement.
* If the particle moved from ***X*** to ***p*** in the time ***t***, then
$\angle pox = \angle MPO = \theta = \omega t$
From the ΔMPO

$$\sin\theta = \sin wt = \frac{OM}{a}$$

$$OM = y = a\sin wt$$

OM is called the displacement of the vibration particle.

* (**The displacement** of a vibrating particle at any instant can be defined as its distance from the mean position of rest.)

Amplitude; the maximum displacements of vibrating particle is called its amplitude.

$$\text{Displacement} = y = a\sin wt$$

* The rate of change of displacement is called the velocity of the vibrating particle.

$$Velocity = \frac{dy}{dt} = aw\cos wt$$

* The rate of change of velocity is called its acceleration.
Acceleration = rate of change of velocity

$$\text{Acceleration} = \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d^2y}{dt^2} = -aw^2\sin wt = -w^2 y$$

Angle	Position of the vibrating particle	Displacement	Velocity	Acceleration
wt	M	$y = a\sin wt$	$\frac{dy}{dt} = aw\cos wt$	$\frac{d^2y}{dt^2} = -aw^2 \sin wt$
0	O	0	+aw	0
$\frac{\pi}{2}$	Y	+a	0	$-aw^2$
π	O	0	-aw	0
$\frac{3\pi}{2}$	Y'	-a	0	$+aw^2$
2π	O	0	+aw	0

Oscillatory behavior;

The process repeats itself periodically. Thus the system Oscillates.
In this process, ***Y, dy/dt, d^2y/dt^2*** Continuously change with respect to time.

[Thus, the velocity of the vibrating particle is maximum at the mean position of rest and zero at the maximum position of vibration].

The acceleration of the vibrating particle is zero at the mean position of rest and maximum at the maximum position of vibration. The acceleration is always directed towards the mean position of rest and is directly proportional to the displacement of the vibrating particle.]

(This type of motion where the acceleration is directed towards a fixed point (the mean position of rest) and is proportional to the displacement of the vibrating particle is called simple harmonic motion)

$$\text{Acceleration} = \frac{d^2y}{dt^2} = -w^2 y = -w^2 \times displacement$$

Numerically $$w^2 = \frac{Acceleration}{Displacement}$$

$$w = 2\pi n = \sqrt{\frac{Acceleration}{Displacement}} = \frac{2\pi}{T}$$

$$T = 2\pi\sqrt{\frac{Displacement}{Acceleration}} = 2\pi\sqrt{K}$$ The time period of a particle vibrating simple harmonically

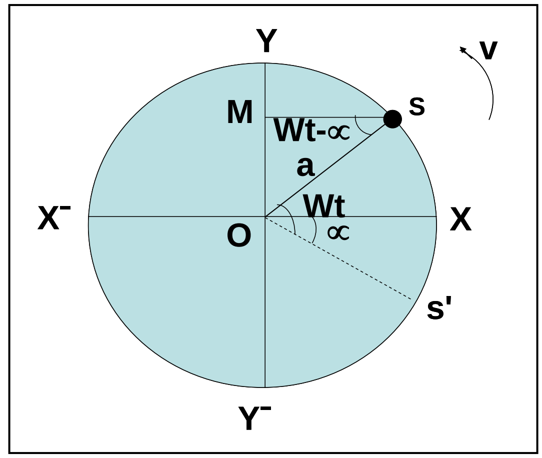
Where K is the displacement per unit acceleration

If the particle ***P*** revolves round the circle, ***n*** times per second, then the angular velocity ***w*** is given by;

$$w = 2\pi n = \frac{2\pi}{T}$$

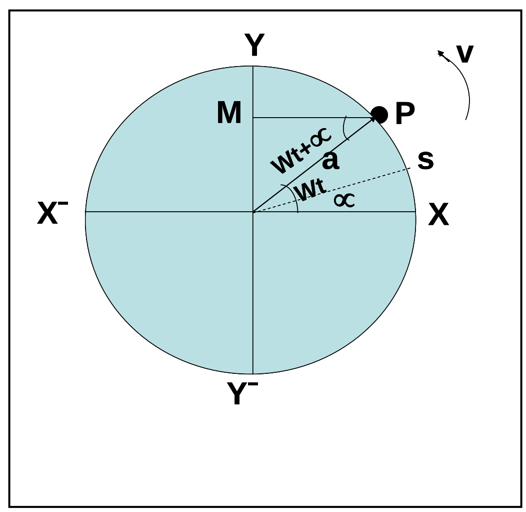
$\therefore n = \dfrac{1}{T}$ Where T is the time period

$$y = a \sin 2\pi n t = a \sin 2\pi \frac{t}{T}$$



On the other hand, if the time is counted
From the instant P is at S ($\angle sox = \propto$)
Then the displacement;

$$y = a \sin(wt + \alpha) = a \sin(2\pi \frac{t}{T} + \alpha)$$



If the time is counted from the instant P is at S' then;

$$y = a \sin(wt - \alpha) = a \sin(2\pi \frac{t}{T} - \alpha)$$

Phase of the vibrating particle:

1- The phase of a vibrating particle is defined as the ratio of the displacement of the vibrating article at any instant to the amplitude of the vibrating particle (y/a).
2- It is also defined as the fraction of the time interval that has lapsed since the particle crossed mean position of rest in the position direction.
3- It is also equal to the angle by the radius vector since the vibrating particle last crossed its mean position of rest e.g., in the above equations wt, (wt+∝) or (wt-∝) are called phase angle.

Thus ∝ is called the epoch in the above expressions.

1.3: Differential Equation of SHM;

For a particle vibrating simple harmonically, the general equation of displacement is,

$$y = a\sin(wt+\alpha)----(1)$$

Where y is displacement, is amplitude and $\propto$ is epoch of the vibrating particle.

Differential Equation (1) with reseat to time

$$\frac{dy}{dt} = a\,w\cos(wt+\alpha)----(2)$$

$\frac{dy}{dt}$ is velocity of the vibrating particle.

Differential Equation (2) with reseat to time

$$\frac{d^2y}{dt^2} = -a\,w^2\sin(wt+\alpha)----(3)$$

But $y = a\sin(wt+\alpha)$

$$\therefore \frac{d^2y}{dt^2} = -w^2 y$$

$$\therefore \frac{d^2y}{dt^2} + w^2 y = 0------(3)$$ Differential Equation of Simple Harmonic Motion

$\frac{d^2y}{dt^2}$ represents the acceleration of the particle.

Equation (3) represents the Differential Equation of Simple Harmonic Motion.

It also shows that in any phenomenon where an equation similar to equation (3) is obtained, the body executes simple harmonic motion.

The general solution of equation (3) is

$$y = a\sin(wt+\alpha)$$

* Also the time period of a vibrating particle can be calculated from equation (3).

Numerically;

$$w = 2\pi n = \sqrt{\frac{Acceleration}{Displacement}} = \sqrt{\frac{d^2y/dt^2}{y}}$$

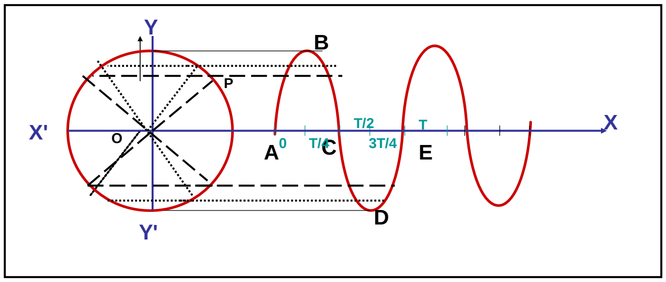
$$T = \frac{2\pi}{w} = 2\pi\sqrt{\frac{Displacement}{Acceleration}}$$

1.4: Graphical Representation of SHM;

Let ***P*** a particle moving on the circumference of a circular of radius ***a***. The foot of the perpendicular vibrates on the diameter YY'.

$$y = a\sin(wt) = a\sin(2\pi \frac{t}{T})$$

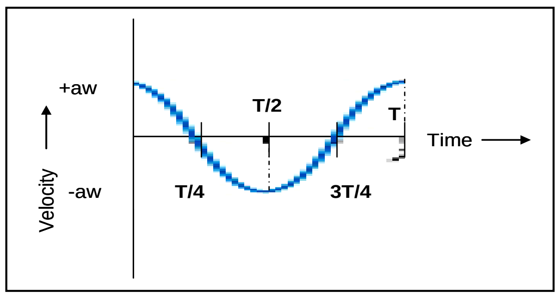
The displacement graph is a sine curve represented by ABCD



The motion of the particle M is SH.

The velocity of a particle moving with SHM is

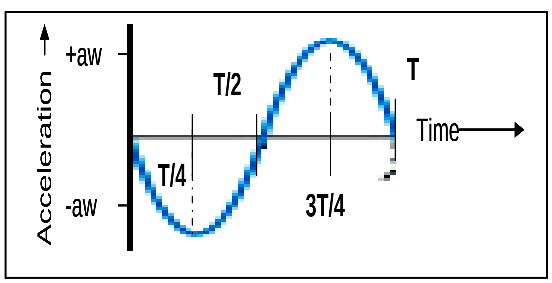
$$v = \frac{dy}{dt} = aw\cos(wt)$$



The velocity – time graph is shown in figure (*) It is cosine curve.

The acceleration of a particle moving with SHM is

$$\frac{d^2y}{dt^2} = -aw^2\sin(wt)$$



The acceleration – time graph is shown in figure (*) It is negative sine curve.

1.5: Average Kinetic Energy of a Vibrating Particle:-

The displacement of a vibrating particle is given by;

$$y = a\sin(wt+\alpha)$$

$$v = \frac{dy}{dt} = a\,w\cos(wt+\alpha)$$

If m is the mass of the vibrating particle, the kinetic energy at any instant

$$K.E = \frac{1}{2}mv^2$$

$$K.E = \frac{1}{2}ma^2w^2\cos^2(wt+\alpha)$$

The average kinetic energy of the particle in one complete vibration

$$\text{Average K.E} = \frac{1}{\text{T}}\int_0^T (K.E)\,dt = \frac{1}{\text{T}}\int_0^T \frac{1}{2}ma^2w^2\cos^2(wt+\alpha)\,dt$$

$$\text{Average K.E} = \frac{1}{\text{T}}\frac{ma^2w^2}{4}\int_0^T 2\cos^2(wt+\alpha)\,dt$$

$$\text{Average K.E} = \frac{1}{\text{T}}\frac{ma^2w^2}{4}\int_0^T (1+\cos 2(wt+\alpha)\,dt$$

$$\text{Average K.E} = \frac{1}{\text{T}}\frac{ma^2w^2}{4}\left(\int_0^T dt + \int_0^T \cos 2(wt+\alpha)\,dt\right)$$

But $\int_0^{\pi} \cos 2(wt+\alpha)\,dt = 0$

$$\therefore \text{Average K.E} = \frac{1}{\text{T}}\frac{ma^2w^2}{4}(T+0)$$

But w=2πn

$$\therefore \text{Average K.E} = \frac{ma^2 4\pi^2 n^2}{4}$$

$$\therefore \text{Average K.E} = \pi^2 ma^2 n^2$$

Where m is the mass of the vibrating particle
a is the amplitude of vibrating
n is the frequency of vibrating

Also, the average K.E of vibrating particle is directly proportional to the square of the amplitude.

$$\text{Average K.E} \propto a^2$$

1.6: Total Energy of Vibrating Particle;

$$y = a\sin(wt+\alpha) \quad \rightarrow \quad \sin(wt+\alpha) = \frac{y}{a} \quad \rightarrow \quad \sin^2(wt+\alpha) = \frac{y^2}{a^2} \qquad \ldots\ldots\ldots(1)$$

$$\because \sin^2(wt+\alpha) + \cos^2(wt+\alpha) = 1 \quad \rightarrow \quad \sin^2(wt+\alpha) = 1 - \cos^2(wt+\alpha) \qquad \ldots\ldots\ldots(2)$$

From eq 1& 2 : $\therefore 1 - \cos^2(wt+\alpha) = \dfrac{y^2}{a^2} \quad \rightarrow \quad \cos^2(wt+\alpha) = 1 - \dfrac{y^2}{a^2}$

$$\cos(wt+\alpha) = \sqrt{1-\frac{y^2}{a^2}} = \sqrt{\frac{a^2-y^2}{a^2}} \quad \rightarrow \quad \cos(wt+\alpha) = \frac{\sqrt{a^2-y^2}}{a} \qquad \ldots\ldots\ldots\ldots(3)$$

From equation $v = \dfrac{dy}{dt} = a\,w\cos(wt+\alpha)$

$$v = \frac{dy}{dt} = a\,w\cos(wt+\alpha) = aw\frac{\sqrt{a^2-y^2}}{a} = w\sqrt{a^2-y^2}$$

$$v = \frac{dy}{dt} = w\sqrt{a^2-y^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

The **K.E** of the particle at the instant the displacement is y,

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}mw^2(a^2-y^2) \qquad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Potential energy of the vibrating particle is the amount of work done in overcoming the force through a distance y.

Acceleration = - w^2y

Force = mass x acceleration = - $m\,w^2y$

(The –Ve sign shows that the direction of the acceleration and force are opposite to the direction of motion of the vibrating particle)

$$P.E = -\int_0^y F\,dy = \int_0^y mw^2y\,dy = mw^2\frac{y^2}{2} = \frac{1}{2}mw^2y^2 \qquad \ldots\ldots\ldots\ldots(6)$$

Total energy of the particle at the instant the displacement is y;

$$E_{Total} = K.E + P.E = \frac{1}{2}mw^2(a^2-y^2) + \frac{1}{2}mw^2y^2$$

$$E_{Total} = \frac{1}{2}mw^2a^2 - \frac{1}{2}mw^2y^2 + \frac{1}{2}mw^2y^2$$

$E_{Total} = \dfrac{1}{2}mw^2a^2$ But w=2πn $E_{Total} = \dfrac{1}{2}m(4\pi^2n^2)a^2$

$$E_{Total} = 2\pi^2mn^2a^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Average K.E of the vibrating particle = $\pi^2mn^2a^2$

Average P.E of the vibrating particle = $\pi^2mn^2a^2$

Total energy at any instant is constant.

Example:

For a particle vibrating simple harmonically, the displacement is 12cm at the instant the velocity is 5cm/sec and the displacement is 5cm at the instant the velocity is 12 cm/sec. Calculate;
(I) Amplitude (II) Frequency and (III) Time Periodic

The velocity of a particle executing SHM

$$v = \frac{dy}{dt} = w\sqrt{a^2 - y^2}$$

In the first case;

$$v_1 = w\sqrt{a^2 - y_1{}^2}$$

v_1 = 5 cm/s, y_1 = 12 cm

$$5 = w\sqrt{a^2 - 144}\ldots\ldots(1)$$

In the second case;

$$v_2 = w\sqrt{a^2 - y_2{}^2}$$

v_1 = 12 cm/s, y_1 = 5 cm

$$12 = w\sqrt{a^2 - 25}\ldots\ldots(2)$$

Dividing (2) by (1) and squaring;

$$\frac{12}{5} = \frac{w\sqrt{a^2 - 25}}{w\sqrt{a^2 - 144}} \qquad \frac{144}{25} = \frac{a^2 - 25}{a^2 - 144}$$

$$144(a^2 - 144) = 25(a^2 - 25) \qquad 144a^2 - 144\times 144 = 25a^2 - 25\times 25$$

$$144a^2 - 20736 = 25a^2 - 625 \qquad 144a^2 - 25a^2 = 20736 - 625$$

$$119a^2 = 20111 \qquad a^2 = \frac{20111}{119} = 169 \qquad \therefore a = 13cm$$

Substituting the value of (a=13cm) in eq.(1);

$$5 = w\sqrt{169 - 144} = w\sqrt{25} = 5w \qquad \therefore w = 1\,radian/\sec$$

The frequency;

$$n = \frac{w}{2\pi} = \frac{1}{2\pi}\,Hertz$$

Time period;

$$T = \frac{1}{n} = 2\pi\,\text{seconds}$$

1.7: Energy of Vibration:

Work done = Force ×displacement

$$y = a\sin(wt-\phi) \quad \text{...............(1)}$$

Let the periodic force be;

$$F = F_o \sin wt \quad \text{...........................(2)}$$

dw=F×dy

The Total Work done;

$w = \int F\,dy$ Since $y = a\sin(wt-\phi) \rightarrow dy = a\cos(wt-\phi)\,d(wt)$

$$w = \int F_o \sin wt\,\big(a\cos(wt-\phi)\big)d(wt)$$

Work done per cycle of motion;

$$w = aF_o \int_0^{2\pi} \sin wt\ \cos(wt-\phi)\,d(wt)$$

But $\cos(wt-\phi) = \cos wt\cos\phi + \sin wt\sin\phi$

Therefore; $w = aF_o \int_0^{2\pi} \sin wt\ \big(\cos wt\cos\phi + \sin wt\sin\phi\big)d(wt)$

$$w = aF_o\cos\phi \int_0^{2\pi} \sin wt\ \cos wt\,d(wt) + aF_o\sin\phi \int_0^{2\pi} \sin^2 wt\,d(wt)$$

$$\int_0^{2\pi} \sin\theta\ \cos\theta\,d\theta = \frac{1}{2}\sin 2\theta$$

$$\int_0^{2\pi} \sin^2\theta\,d\theta = \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)$$

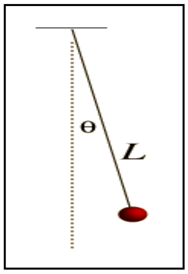
$$w = aF_o\cos\phi\left(\frac{\sin 2wt}{2}\right)\Big|_0^{2\pi} + aF_o\sin\phi\left(\frac{wt}{2} - \frac{\sin 2wt}{4}\right)\Big|_0^{2\pi}$$

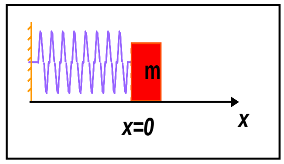
$$w = aF_o\sin\phi(\pi)$$

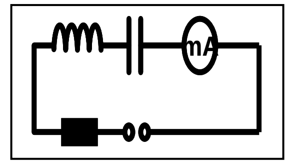
Or

$$w = \pi aF_o\sin\phi$$

1.8: Oscillation with One Degree of Freedom;







In the case of simple pendulum **Loaded spring** **LC circuit**

These oscillations take place about the mean position. All these systems have one degree of freedom.

For oscillatory with one degree of freedom, the displacement the "Moving particles" depends upon the SHM

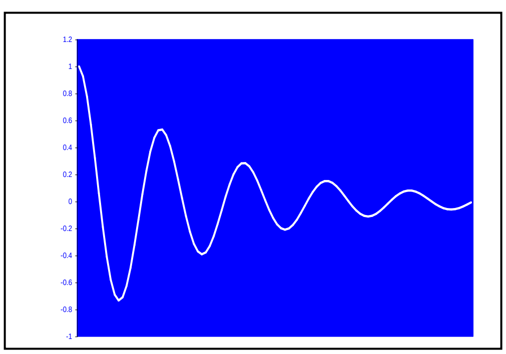
$$y = a\sin(wt + \alpha)$$

An oscillatory system will continue to oscillate for an infinite time according to the equation $y = a\sin(wt + \alpha)$.

Damped Oscillations; In actual particle the oscillatory system experiences fractional or resistive forces. Due to these reasons the oscillations get damped.

In the case of pendulum, the amplitude decreases due to the resistance offered by air.

In the case of LC circuit, the resistance of the circuit produces damped.



1.9: Linearly and Superposition Principles:

The differential of SHM is given by;

$$\frac{d^2y}{dt^2}+w^2y=0$$

In this equation;

$\frac{d^2y}{dt^2}$ is proportional to –y

The time periods for a simple pendulum
Mass and spring
Katter's pendulum
LC circuit and etc….

are derived on the basis of this equation.
In all these case, the return force per unit displacement depends upon the term y.
The equation does not contain the terms y^2, y^3 etc.
The differential equations which do not contain the higher powers of y, such as y^2, y^3 ….etc. terms are called **Linear equations.**

Further this equation does not contain any term independent of y. therefore; it is called a linear homogeneous equation.
If, in a particular equation higher power of y (i.e. y^2, y^3 …..etc. terms are present, the equation is said to be **non-linear**.
Moreover, if it contains terms independent of y also, it is said to be **non-homogeneous** and **non-linear** equation.

In general, it is not easy to solve non-linear equations.

The equation for a simple pendulum is

$$M\frac{d^2y}{dt^2}=-Mg\sin\theta$$

It is assumed that amplitude is small and sinθ=θ but, $\sin\theta=\theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}-........$

$$M\frac{d^2y}{dt^2}=-Mg\theta \quad ……………..(1)$$

is **linear**

Taking this value of sinθ

$$M\frac{d^2y}{dt^2}=-Mg\left(\theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}-.......\right) \quad …………….(2)$$

is non-linear for it contains terms of θ^3, θ^5 *etc.*

Linear homogeneous equations:

* One of the important properties of linear homogeneous equations is that the sum of any two solutions is a solution by itself. This property is not true in the case of non-linear equation.

Consider the differential equation;

$$\frac{d^2y}{dt^2} = -w^2y + Ay^2 + By^3 + Cy^4 + ...etc - - - - - (3)$$

If the value of the constant A,B,C….etc in eq.(3) are Zero or they can be taken sufficiently near to Zero, then equation (3) becomes;

$$\frac{d^2y}{dt^2} = -w^2y - - - - - - (4)$$ This equation is linear & homogeneous

If A,B,C….etc are not Zero, then equation is non-linear.

Suppose, y_1 is the first solution of eq.(3) at some instant of time t_1

Suppose, y_2 is the second solution of eq.(3) at some instant of time t_2

(Differential displacement & velocity)

$$\therefore \frac{d^2y_1}{dt_1^{\,2}} = -w^2y_1 + Ay_1^{\,2} + By_1^{\,3} + Cy_1^{\,4} + ...etc - - - - - (5)$$

$$and\ \frac{d^2y_2}{dt_2^{\,2}} = -w^2y_2 + Ay_2^{\,2} + By_2^{\,3} + Cy_2^{\,4} + ...etc - - - - - (6)$$

When the two instant are superimposed on each other, the resultant displacement is y.

$Y = y_1 + y_2$

∴ The equation for resultant displacement;

$$\frac{d^2y}{dt^2} = \frac{d^2}{dt^2}(y_1 + y_2) = -w^2(y_1 + y_2) + A(y_1 + y_2)^2 + B(y_1 + y_2)^3 + C(y_1 + y_2)^4 + ... - - (7)$$

Adding equations (5) and (6);

$$\frac{d^2y_1}{dt_1^{\,2}} + \frac{d^2y_2}{dt_2^{\,2}} = -w^2(y_1 + y_2) + A(y_1^{\,2} + y_2^{\,2}) + B(y_1^{\,3} + y_2^{\,3}) + C(y_1^{\,4} + y_2^{\,4}) + ... - - - - - - (8)$$

The equations (7) and (8) are identical, only if

$$\frac{d^2}{dt^2}(y_1 + y_2) = \frac{d^2y_1}{dt^2} + \frac{d^2y_2}{dt^2} - - - - (9) \rightarrow -w^2(y_1 + y_2) = -w^2y_1 - w^2y_2 - - - - - (10)$$

$$A(y_1^{\,2} + y_2^{\,2}) = A(y_1 + y_2)^2 - - - - - - (11) \rightarrow B(y_1^{\,3} + y_2^{\,3}) = B(y_1 + y_2)^3 - - - - - (12)$$

$$C(y_1^{\,4} + y_2^{\,4}) = C(y_1 + y_2)^4 - - - - - - (13)$$

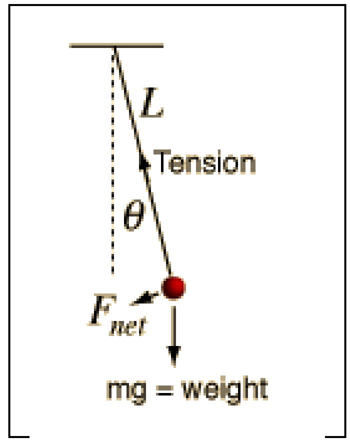
Equations (9) and (10) are true, But equations (11),(12) and (13) are true only, A=0,B=0, and C=0.

When A,B,C,….etc are Zero, the equation become linear. Hence superposition principle is true only in the case of homogeneous linear equation.

- Also the sum of any two solutions is also a solution of the homogenous linear equation.
- All harmonic oscillators given in equation (9) and (10) obey superposition principle.

1.11: Simple Pendulum:

A Simple Pendulum of a light string supporting a small sphere and fixed firmly at its upper end, an ideal Simple Pendulum should consist of a heavy particle suspended by means of a weight less, inextensible, flexible string from a rigid support.



Let pendulum be displaced from its mean position O and allowed to oscillate.
Suppose at any instant of time t, it is at A.
The force acting upon the polo vertically downward=***Mg***
Resolve ***Mg*** into rectangular components:

1. Force along the string =***Mg Cos*** θ
2. Force perpendicular to the string = ***Mg Sin*** θ

Let the tension in the string be ***T***. The component ***Mg Cos*** θ balances the tension T.

$$Mg\,Cos\theta = T \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Thus the only force acting on the oscillating particle is ***–Mg Sin*** θ

$$F = -Mg\,Sin\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$\therefore$ (-Ve sign shows that the acceleration is directed towards the mean pos

According to Taylor's series of expansion; $Sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \ldots\ldots\ldots$

For small angular displacement θ; $Sin\theta = \theta$

$\therefore$ Tangential force, $F = -Mg\theta$ ………………..(3)

The linear displacement, y=ℓθ $\rightarrow \frac{dy}{dt} = l\frac{d\theta}{dt}$ $\rightarrow$ Acceleration, $\frac{d^2y}{dt^2} = l\frac{d^2\theta}{dt^2}$

$\therefore$ Force, $F = ma' = Ml\frac{d^2\theta}{dt^2}$ …………………(4) From Newton's second law.

From eq 3&4 ; $\cancel{M}l\frac{d^2\theta}{dt^2} = -\cancel{M}g\theta \rightarrow \frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta$

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0 ------(5)$$

The eq. is similar to the eq. of SHM

$$\frac{d^2y}{dt^2} + w^2\theta = 0 ------(6)$$

From equations (5) and (6);

$$w^2 = \frac{g}{l} \qquad w = \sqrt{\frac{g}{l}}$$

Time period, $T = \frac{2\pi}{w}$ $\quad \therefore T = 2\pi\sqrt{\frac{l}{g}}$

In case of a simple pendulum
If the size of the bob is large, a correction to be applied

$$\therefore t = 2\pi\sqrt{\frac{l + (\frac{2}{5}r^2)/l}{g}}$$

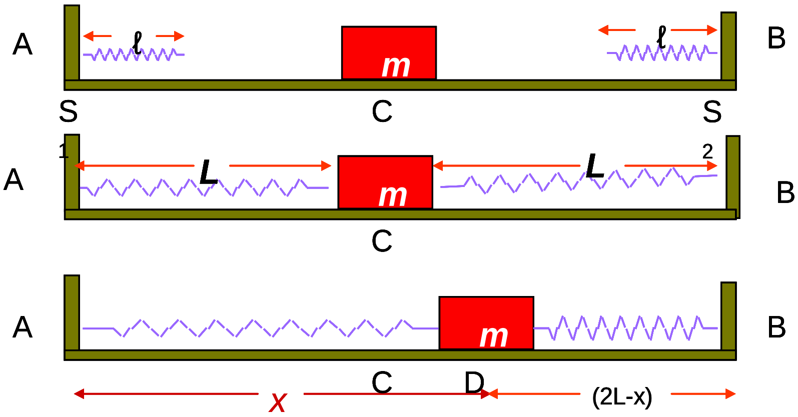
Here $l + (\frac{2}{5}r^2)/l$ represent the equivalent Length of a simple pendulum.

1.12: Simple Harmonic Oscillation of a Mass between two springs:-

Consider two spring S1 and S2 each having a length l in the relaxed position.

Mass M is placed midway between the two springs on a frictionless surface.

One end of the spring S1 is attached to a rigid wall A and the other end is attached to the Mass M. Similarly one end of the spring S2 is attached to a rigid wall at B and the other end is connected to the mass M.



Here AC=BC=L

At C the mass is equally pulled by both the springs and it is the equilibrium position.

When the mass M is displaced from its equilibrium position and left, it excites SHO.

Let, at any instant, D be the displaced position of the mass M.

Here AD=x, and BD=(2L-x)

Let the tension per unit displacement in the spring be K.

- The displacement of the spring of S1 is (x-L) and it extents a force =K(x-L) in the direction DA.
- * The displacement of the spring of S2 is (2L-x-L) and it extents a force =K(2L-x-L) in the direction DB.

The resultant force on the mass M= K(2L-x-L)- K(x-L) in the direction DB

= 2Lk-xk-Lk-kx+kL=2Lk-2xk=2k(L-x)

= -2k (x-L) in the direction DB

According to Newton's second law of motion;

$$F = M\frac{d^2x}{dt^2} = -2k\,(x - L) - - - - - - - (1)$$

$$\therefore \frac{d^2x}{dt^2} = -\frac{2k}{M}(x - L)$$

$$or \ \frac{d^2x}{dt^2} + \frac{2k}{M}(x - L) = 0 \text{-------} (2)$$

Taking the displacement from the mean position;

$x\text{-}L=y \rightarrow x=y+L$

Differentiating twice, $\frac{dx}{dt} = \frac{dy}{dt} \rightarrow \frac{d^2x}{dt^2} = \frac{d^2y}{dt^2}$

Substituting these value in eq.(2);

$$\frac{d^2y}{dt^2} + \frac{2k}{M} y = 0 \text{-------} (3)$$

This equation is similar to the equation of SHM;

$$\frac{d^2y}{dt^2} + w^2 y = 0 \text{-------} (4)$$

From eqs(3) and (4);

$$w^2 = \frac{2k}{M}$$

$$w = \sqrt{\frac{2k}{M}}$$

Time Period, $T = \frac{2\pi}{w} = 2\pi\sqrt{\frac{M}{2k}} \quad ------(5)$

End Chapter One