Chapter Two
Lissajous' Figures

2.1: Composition of Two Simple Harmonic Motion in a Straight Line:

- **Analytical method;-** Let the two simple harmonic vibrations be represented by the equations;

$$y_1 = a_1 \sin(wt + \alpha_1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$y_2 = a_2 \sin(wt + \alpha_2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$Y = y_1 + y_2$$

$$\begin{aligned}
y &= a_1 \sin(wt + \alpha_1) + a_2 \sin(wt + \alpha_2) \\
&= a_1 \left(\sin wt \cos \alpha_1 + \cos wt \sin \alpha_1\right) + a_2 \left(\sin wt \cos \alpha_2 + \cos wt \sin \alpha_2\right) \\
&= a_1 \sin wt \cos \alpha_1 + a_1 \cos wt \sin \alpha_1 + a_2 \sin wt \cos \alpha_2 + a_2 \cos wt \sin \alpha_2 \\
&= (a_1 \sin wt \cos \alpha_1 + a_2 \sin wt \cos \alpha_2) + (a_1 \cos wt \sin \alpha_1 + a_2 \cos wt \sin \alpha_2) \\
&= (a_1 \cos \alpha_1 + a_2 \cos \alpha_2) \sin wt + (a_1 \sin \alpha_1 + a_2 \sin \alpha_2) \cos wt \quad \ldots\ldots\ldots\ldots(3)
\end{aligned}$$

$$A\cos\varphi = a_1\cos\alpha_1 + a_2\cos\alpha_2 \;\ldots\ldots\ldots\; (4)$$

$$A\sin\varphi = a_1\sin\alpha_1 + a_2\sin\alpha_2 \;\ldots\ldots\ldots\; (5)$$

$$A_2 = a_1^2 + a_2^2 + 2a_1a_2\cos(\alpha_1 - \alpha_2) \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; (6)$$

$$\tan\varphi = \frac{a_1\sin\alpha_1 + a_2\sin\alpha_2}{a_1\cos\alpha_1 + a_2\cos\alpha_2}\ldots\ldots\ldots\ldots(7)$$

$$y_1 = A\cos\varphi\sin wt + A\sin\varphi\cos wt$$

$$y = A\sin(wt + \varphi)\ldots\ldots\ldots(8)$$

2.2: Composition of Two Simple Harmonic Vibrations of Equal Time Periods Acting at Right Angle:-

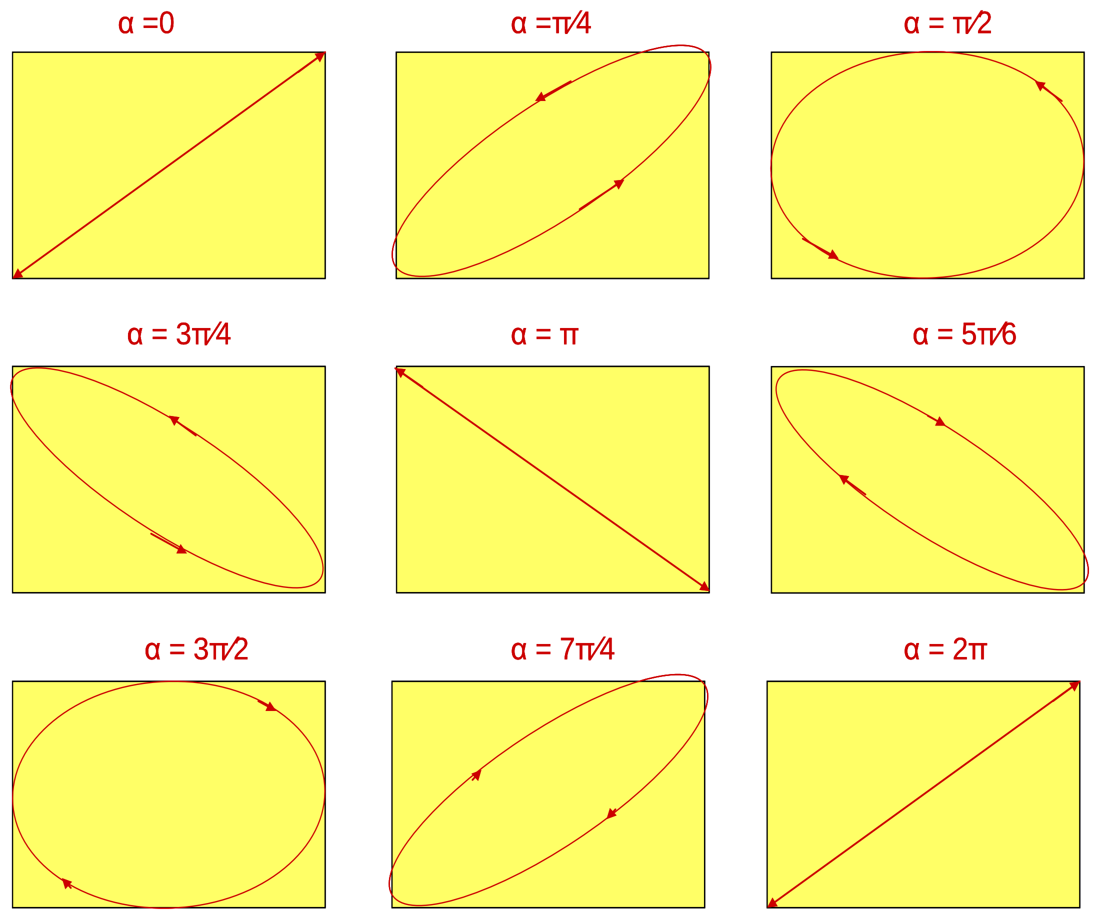
$$x = a\ \sin(wt + \alpha) ....(1)$$

$$y = b\ \sin wt ....(2)$$

$$\frac{x}{a} = \sin wt \cos\alpha + \cos wt \sin\alpha ......(3)$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\alpha = \sin^2\alpha ....(4)$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy}{ab}\cos\alpha=\sin^2\alpha.................(4)$$



2.3: Composition of Two Simple Harmonic Motion at Right Angle to Each Other and Having Time Period in the ratio1:2

$$x = a\,\sin(2wt + \alpha)\text{.......}(1)$$

$$y = b\,\sin wt\text{..................}(2)$$

$$\frac{x}{a} = \sin 2wt\cos\alpha + \cos 2wt\sin\alpha$$

$$= 2\sin wt\cos wt\cos\alpha + \left(1 - 2\sin^2 wt\right)\sin\alpha$$

$$\left(\frac{x}{a} - \sin\alpha\right)^2 + \frac{4y^2}{b^2}\left[\frac{y^2}{b^2} + \frac{x}{a}\sin\alpha - 1\right] = 0\text{.........................}(3)$$