Chapter Four

Wave Motion

Wave Motion

Introduction:

- When a stone is drooped into a pond containing water, Waves are produced at the point where the stone strikes the water in the pond.
- The wave travels from one particle to the next but the particles of air vibrate about their mean position.
- Wave motion in general, refers to the transfer of energy from one point to another point of the medium.
- Transference of various form of energy like **sound**, **light**, **x-ray**, **γ-ray**, **radio waves**; etc, takes place in the form of wave motion.

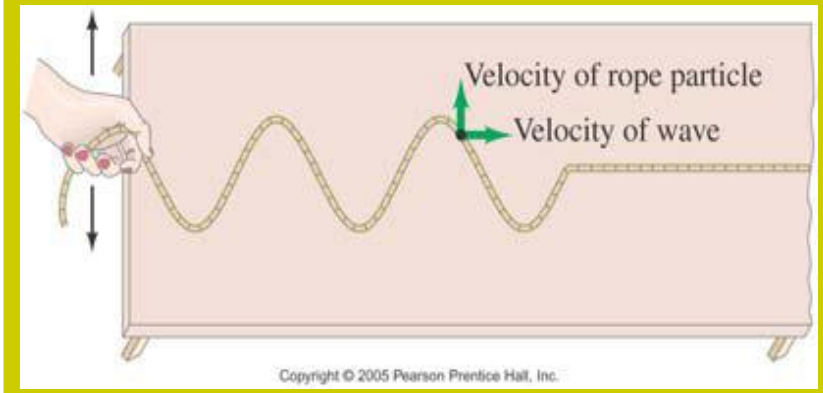
4.1: Characteristic of Wave Motion:

1-Wave motion is a disturbance produced in the medium by the repeated periodic motion of the particles of the medium.

2-Only the wave travels forward whereas the particles of the medium vibrate about their mean position.

3-There is a regular phase change between the various particles of the medium. The particle a head starts vibrating a little later than a particle just preceding it.

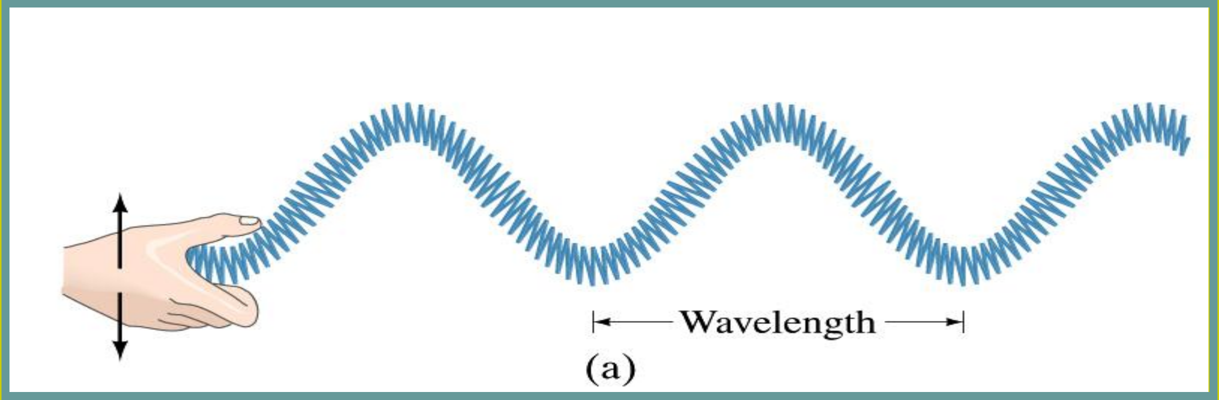
4-The velocity of the wave is different from the velocity with which the particles of the medium are vibrating about their mean position. The wave travels with a uniform velocity whereas the velocity of the particles is different position. It is maximum at the mean position and zero at the extreme position the particles.



A wave travels along its medium, but the individual particles just move up and down.

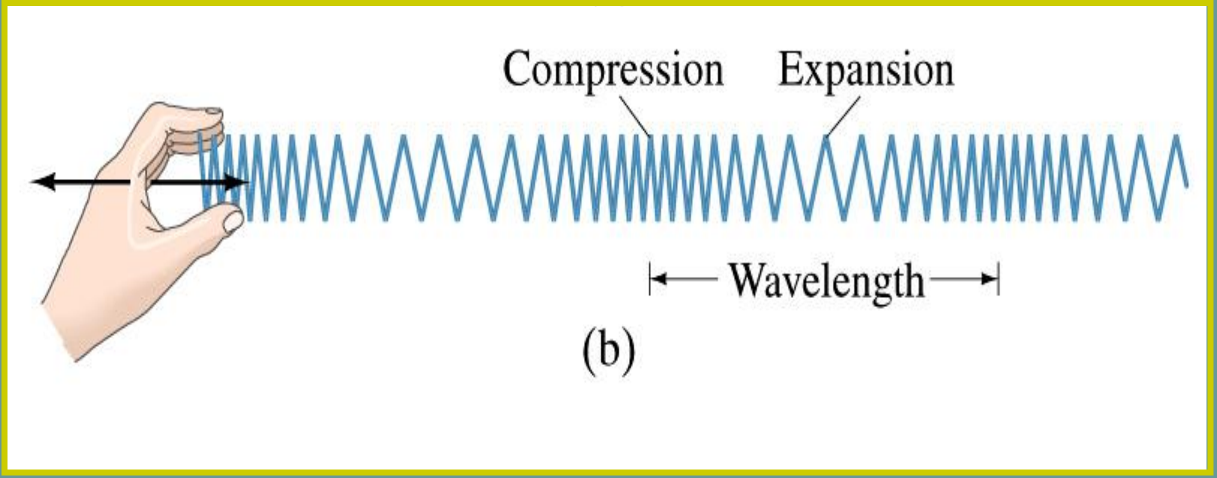
4.2: Transverse Wave Motion:

- In this type of wave motion, the particles of medium vibrate at right angles to the direction of the wave. Show the figure (4-1-a).



4.3: Longitudinal Wave Motion:

- In this type of wave motion, particles of medium vibrate along the direction of propagation of the wave. Show the figure (4-1-b).

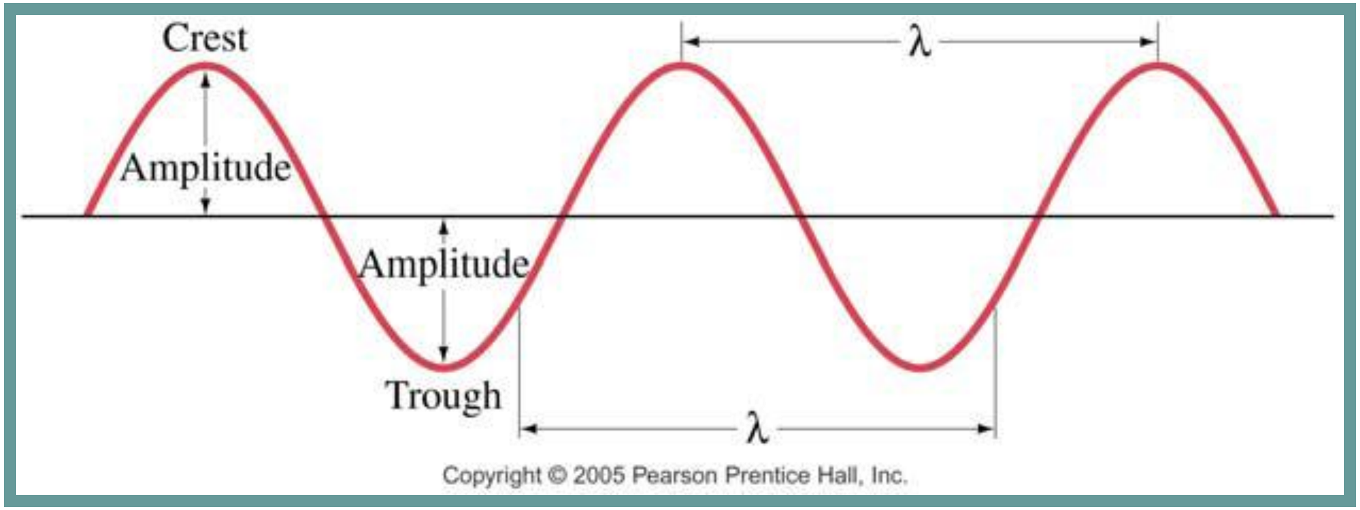


4.4: Definitions:

- **Wave length:** It is the distance traveled by the wave in the time in which the particle of the medium completes one vibration. It is also defined as the distance between two nearest particles in the same phase.
- **Frequency:** It is the number of vibrations made by a particle in one second.
- **Amplitude:** It is the maximum displacement of the particle from its mean position of rest.
- **Time Period:** It is the time taken by a particle to complete one vibration.

∴ ***Frequency × Time period = 1***

- **Vibration:** It is the to-and-for motion of a particle from one extreme position to the other and back again. It is also equal to the motion of a particle from the mean position to one extreme position, then to the other extreme position and finally back to the mean position.
- **Phase:** It is defined as the ratio of the displacement of the vibrating particle at any instant to the amplitude of the vibrating particle or it is defined as the fraction of the time interval that has elapsed since the particle crossed the mean position of rest in the positive direction or it is also equal to the angle swept by the radius vector since the vibrating particle last crossed its mean position of rest.



4.5: Relation between Frequency and Wavelength:

- **Velocity of the wave is the distance traveled by the wave in one second.**

$$Velocity = \frac{Dis\tan ce}{Time}$$

- **Wavelength (λ) is the distance traveled by the wave in one time period (T).**

$$Velocity = \frac{Wavelength}{Time\ Period} = \frac{\lambda}{T}$$

$T = 1/n$

, *Frequency × Time period* =

$$v = \frac{\lambda}{T} = \frac{\lambda}{\frac{1}{n}}$$

$$\therefore v = n\lambda$$

4.6: Properties of Longitudinal Progressive Waves:

- 1- The particle of the medium vibrate simple harmonically along the direction of propagation of the wave.
- 2- All the particle has the same amplitude, frequency and time period.
- 3- There is a gradual phase difference between the successive particles.
- 4- All particles vibrating in phase will be at a distance equal to nλ. Here n=1, 2, 3, etc. It means the minimum distance two particles vibrating in phase is equal to the wavelength.
- 5- The velocity of the particle is a maximum at their mean position and it is zero at their extreme position.
- 6- When the particle moves in the same direction as the propagation of the wave, it is in a region of compression.
- 7- When the particle moves in a direction opposite to the direction of propagation of the wave, it is in a region of rarefaction.
- 8- When the particle is at the mean position, it is a region of maximum compression or rarefaction.
- 9- When the particle is at the extreme position, the medium around the particle has its normal density, with compression on one side and rarefaction on the other.
- 10- Due to repeated periodic motion of the particles, compressions and rarefaction are produced continuously. These compressions and rarefactions travel forward along the wave and transfer energy in the direction of propagation of the wave.

4.7: Equation of a Simple Harmonic Wave:

- Consider a particle O in a medium. Let the displacement at any instant of time be given by;

$$y = a\sin wt ------(1)$$

- Consider another particle A at a distance x from the particle O to its right. Here, it is assumed that the wave is traveling with a velocity from left to right i.e., from particle towards A.

$$y = a\sin(wt - \alpha) ------(2)$$

- Where ∝ is the phase difference between the particle O and A. For a phase difference of 2π, the path difference is λ. Suppose for a phase difference of ∝, the path difference is x.

$$\therefore \frac{\alpha}{x} = \frac{2\pi}{\lambda} \qquad \therefore \alpha = \frac{2\pi x}{\lambda} \qquad w = \frac{2\pi v}{\lambda}$$

$$y = a\sin\left(\frac{2\pi v}{\lambda}t - \frac{2\pi x}{\lambda}\right)$$

$$y = a\sin\frac{2\pi}{\lambda}(vt - x) ------(3)$$

$$y = a\sin\frac{2\pi}{\lambda}(vt + x) ------(4)$$

4.8: Differential Equation of Wave Motion:

$$y = a\sin\frac{2\pi}{\lambda}(vt - x) ------(1)$$

$$\frac{dy}{dt} = \frac{2\pi v}{\lambda} a\cos\frac{2\pi}{\lambda}(vt - x) ------(2)$$

$$\frac{d^2y}{dt^2} = -\frac{4\pi^2 v^2}{\lambda^2} a\sin\frac{2\pi}{\lambda}(vt - x) ------(3)$$

$$\frac{dy}{dx} = -\frac{2\pi}{\lambda} a\cos\frac{2\pi}{\lambda}(vt - x) ------(4)$$

$$\frac{d^2y}{dx^2} = -\frac{4\pi^2}{\lambda^2} a\sin\frac{2\pi}{\lambda}(vt - x) ------(5)$$

$$\frac{dy}{dt} = v\frac{dy}{dx} ------(6)$$

$$\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2} ------(7)$$

$$\frac{d^2y}{dt^2} = K\frac{d^2y}{dx^2}$$

$$K = v^2$$

4.9: Particle Velocity & Wave Velocity:

$$y = a\sin\frac{2\pi}{\lambda}(vt - x) ------(1)$$

$$u = \frac{dy}{dt} = \frac{2\pi a v}{\lambda}\cos\frac{2\pi}{\lambda}(vt - x) ------(2)$$

$$u_{\max} = \frac{2\pi a v}{\lambda} ----(3)$$

[Wave velocity]= $\frac{2\pi a}{\lambda}$ [Maximum Particle Velocity]

$$\frac{d^2y}{dt^2} = -\frac{4\pi^2 a v^2}{\lambda^2}\sin\frac{2\pi}{\lambda}(vt - x) ------(4)$$

$$\frac{d^2y}{dt^2} = -\frac{4\pi^2 v^2}{\lambda^2}\left(a\sin\frac{2\pi}{\lambda}(vt - x)\right) = -\left(\frac{4\pi^2 v^2}{\lambda^2}\right)y$$

$$\therefore \frac{d^2y}{dt^2} = - = -\left(\frac{4\pi^2 v^2}{\lambda^2}\right)a ----(5)$$

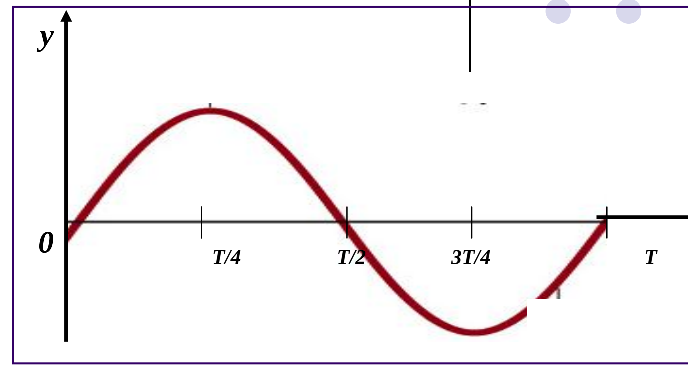
$$\frac{dy}{dx} = -\frac{2\pi a}{\lambda}\cos\frac{2\pi}{\lambda}(vt - x) ------(6)$$

$$u = \frac{dy}{dt} = -v\frac{dy}{dx} ------(7)$$

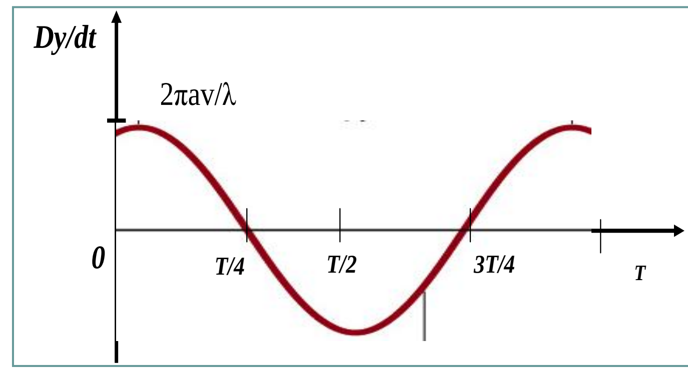
[Particle Velocity at any instant] = [Wave Velocity]x[Slope of the displacement curve at that instant]

4.10: Distribution of Velocity & Pressure in a Progressive Wave:

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) ------(1)$$



$$u = \frac{dy}{dt} = \frac{2\pi a v}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x) -----(2)$$



The strain in the medium is $\frac{dy}{dx}$

$$\frac{dy}{dx} = -\frac{2\pi a}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x)$$

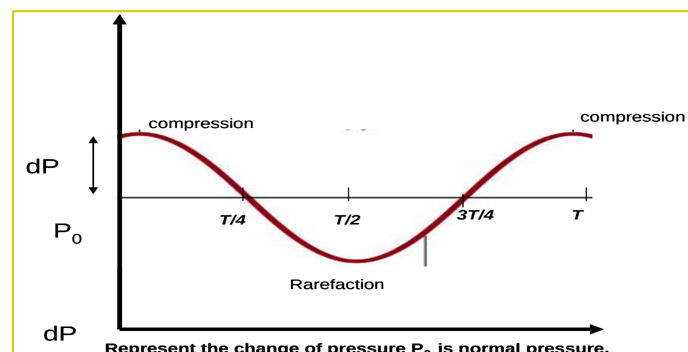
$$K = \frac{Change\ in\ pressure}{Volume\ strain} = \frac{-dP}{dy/dx}$$

$$dP = -K\,(dy/dx)$$

$$dP = K.\left(\frac{-dy}{dx}\right) ------(3)$$

It means if (dy/dx) is negative, dP is +ve, it is region of compression, If (dy/dx) is positive, dP is –ve, i.e., it is a region of rarefaction.

$$dP = K \frac{2\pi a}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x) ------(4)$$



4.11: Energy of a Progressive Wave:

Wave motion in general, refers to the transfer of energy from one point to another point of the medium.