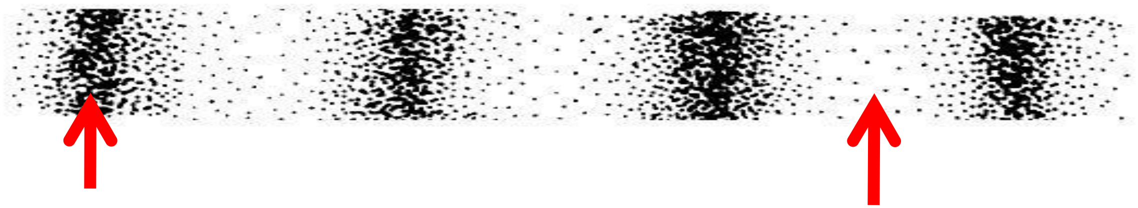
Chapter Five

Velocity of Sound

5.1: Origin of Sound:

- Sound is produced by a vibrating body
- The bell is set into vibration and sound is propagated through air. These vibrations reach the ear and the eardrum is set into vibration. These vibrations are communicated to the brain.
- 

High pressure

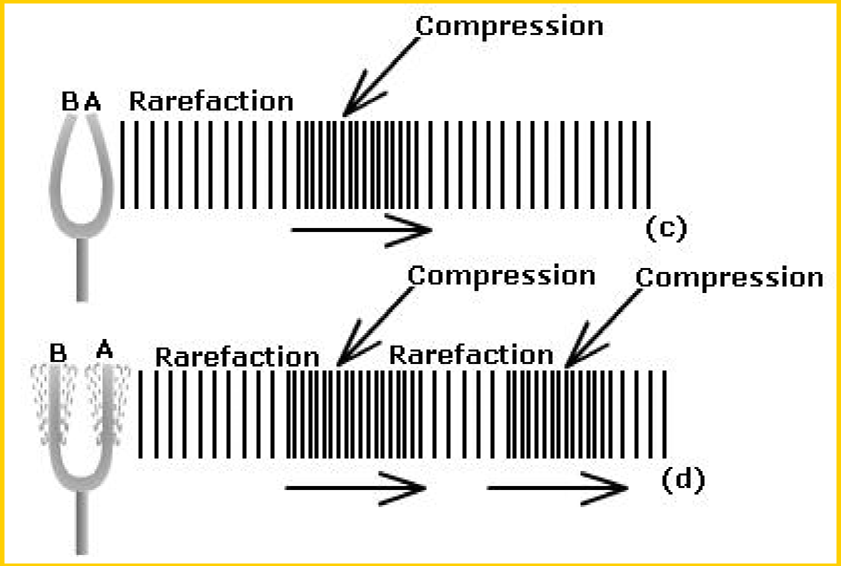
"COMPRESSION"

Low pressure

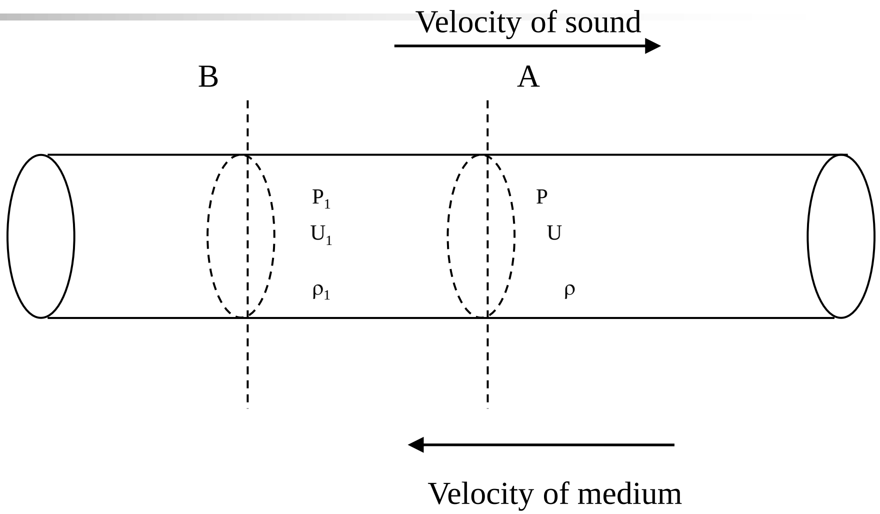
"RAREFACTION"

5.2: Material Medium is a Necessity:

- It can be proved by means of an experiment that a material medium is a necessity for the propagation of sound waves. In the absence of a medium, no sound waves can travel.



5.3: Velocity of Longitudinal Waves in Gases:



Mass of the medium entering the cross-section at (A) in one second $= a \times U \times \rho$

Mass of the medium entering the cross-section at (B) in one second $= a \times U_1 \times \rho_1$

$$\therefore \; m = a\,U\,\rho = a\,U_1\,\rho_1$$

$$\rho < \rho_1 \qquad U > U_1$$

Change in momentum per second = mU – mU$_1$.

momentum per second is due to difference in pressure (P_1 - P) between A and B.

$F=(P_1-P)\,a.$

As there is no change in the average density of the medium, the masses of the medium crossing the sections (A) and (B) in one second are equal.

$$(P_1 - P)\,a = mU - mU_1$$

$$(P_1 - P)\,a = mU\left(1 - \frac{U_1}{U}\right) \quad ---(1)$$

$$\frac{U_1}{U} = \frac{\rho}{\rho_1}$$

$$U^2 \rho = \frac{(P_1 - P)}{\left(\frac{\rho_1 - \rho}{\rho_1}\right)} \qquad ---(2)$$

Substituting the value of m = a Uρ, and

5.3: Velocity of Longitudinal Waves in Gases:

$$(P_1 - P)a = mU(1 - \frac{U_1}{U}) \quad ---(1)$$

$$U^2 \rho = \frac{(P_1 - P)}{(\frac{\rho_1 - \rho}{\rho_1})} \quad ---(2)$$

The bulk modulus of elasticity of the medium;

$$E = \frac{(P_1 - P)}{(\frac{V - V_1}{V})}$$

$$\because V \propto \frac{1}{\rho}$$

$$E = \frac{(P_1 - P)}{(\frac{\rho_1 - \rho}{\rho_1})} \quad ---(3)$$

$$U^2 \rho = E$$

$$U^2 = \frac{E}{\rho} \quad or \quad U = \sqrt{\frac{E}{\rho}}$$

5.4: Newton's Formula for Velocity of Sound:

The velocity of sound in a medium-solid, liquid or gas depends on the elasticity and density of the medium.

$$U = \sqrt{\frac{E}{\rho}}$$

Suppose, initial pressure = P
Initial volume = V
Increase (Change) in pressure = p
Decrease (Change) in volume = ʊ
Final pressure = (P+p)
Final volume = (V-ʊ)
Therefore, (P+p) (V-ʊ) = PV

pV = P ʋ or P=pV/ ʋ ----- (1)

$$E = \frac{Change\ in\ pressure}{Change\ in\ volume/Original\ volume}$$

$$E = \frac{P}{\upsilon/V} = \frac{PV}{\upsilon} \quad ----(2)$$

$$U = \sqrt{\frac{P}{\rho}} \quad ---(3)$$

$$U = \sqrt{\frac{0.76\times136\times10^3\times9.81}{1.293}} = 280\,m/\sec$$

Laplace correction

- The process is not <u>isothermal</u> but it is <u>adiabatic</u>. The total quality of heat of the system as a whole remains constant. It neither gains nor loses any heat to the outside. During an adiabatic process;
- PV^{γ}=constant
- $PV^{\gamma} = (P+p)(V-\upsilon)^{\gamma}$

$$\frac{p}{P} = \frac{\gamma\,\upsilon}{V} \quad or \quad \gamma\,P = \frac{pV}{\upsilon} \quad ----(1)$$

$$E = \frac{pV}{\upsilon} \quad -----(2)$$

$$U = \sqrt{\frac{E}{\rho}}$$

$$U = \sqrt{\frac{\gamma\,P}{\rho}}$$

$$U = \sqrt{\frac{\gamma\,P}{\rho}} = \sqrt{\frac{1.42\times 0.76\times 13.6\times 10^{3}\times 9.81}{1.293}} = 332.8\,m/\sec$$

This value is in agreement with the experimental value.
Hence the correct formula for the velocity of sound in air or a gas is;

$$U = \sqrt{\frac{\gamma\,P}{\rho}}$$

5.5: Effect of Temperature:

- The density of air or a gas changes with the change in temperature.

At T=0°c

$$U_0 = \sqrt{\frac{\gamma P}{\rho_0}} \quad ---(1)$$

At T=t°c

$$U_t = \sqrt{\frac{\gamma P}{\rho_t}} \quad ---(2)$$

$$\frac{U_t}{U_o} = \frac{\sqrt{\frac{\gamma P}{\rho_t}}}{\sqrt{\frac{\gamma P}{\rho_o}}} = \sqrt{\frac{\rho_o}{\rho_t}} = \sqrt{1 + \frac{1}{273}} = \sqrt{\frac{T}{T_{o\,t}}}$$

$$\therefore \quad U \propto \sqrt{T}$$

5.6: Effect of Pressure:

- **Velocity of sound in air is independent of change of pressure.**

$$Volume = \frac{mass}{density} = \frac{m}{\rho}$$

$$PV = \frac{P\,m}{\rho} = cons\tan t$$

$$U = \sqrt{\frac{\gamma P}{\rho}}$$

Since, the mass remains constant, P/ρ is constant. ρ

$$\sqrt{\frac{P}{\rho}}$$

Is constant

Therefore, the velocity of sound in air or a gas is independent of changes in pressure provided the temperature remains constant.

5.7: Effect of Density of the Medium:

Consider two media, of densities ρ_1 and ρ_2

$$U_1 = \sqrt{\frac{\gamma P}{\rho_1}} \quad ---(1)$$

$$U_2 = \sqrt{\frac{\gamma P}{\rho_2}} \quad ---(2)$$

$$\frac{U_2}{U_1} = \frac{\sqrt{\frac{\gamma P}{\rho_1}}}{\sqrt{\frac{\gamma P}{\rho_1}}} = \sqrt{\frac{\rho_1}{\rho_2}}$$

$$or \quad U = \frac{1}{\sqrt{\rho}} \quad ---(3)$$

Velocity of sound in air or a gas is inversely proportional to the square root of the density of the medium.

5.8: Effect of Humidity:

- The density of vapor at NTP=18/22.4=0.8 Kg/m3 whereas the density of dry air at NTP=1.293 Kg/m^3. Therefore, water vapor has air is density less than the density of the dry air. Consequently the density of moist air is less than the density of dry air. As the velocity of sound is more in a medium of lesser density. Velocity of sound in moist air more than the velocity of sound in dry air.

5.9: Effect of Wind:

In the wind is blowing in the direction of propagation of the wave, it will increase the velocity of sound. On the other hand, if the wind is blowing in a direction opposite to the direction of propagation of the sound waves, it will decrease the velocity of sound in air.

5.10: Velocity of Sound in Water:

According to Newton's formula, the velocity of sound in water is given by;

$$U = \sqrt{\frac{E}{\rho}}$$

$$U = \sqrt{\frac{2.23 \times 10^{9}}{10^{3}}} = 10^{3}\ \frac{kg}{m^{2}} = 1493\, m/\sec$$

The velocity of sound in water is approximately four times the velocity of sound in air.

5.12: Velocity of Sound in Isotropic Solids:

- Let one end of the rod with a hammer with s force F for a small interval of time dt.
- Impulse = F dt ----- (1)
- This impulse produces a compressional wave. The distance traveled by the wave in time dt = Udt.
- The particle velocity of the element =dx/dt
- Mass gain in momentum due to the impulse =mass × velocity
- =a × Udt × ρ × (dx/dt)
- = aUρdx ----- (2)
- Impulse = Change in momentum
- Therefore, from equations (1) and (2)
- Fdt= aUρdx ----- (3)
- If Y be the Young's modulus of the material,

Y=stress/strain

$$Y = \frac{F/a}{a\,dx / a\,U\,dt}$$

$$Fdt = Ya\frac{dx}{U} \quad ---(4)$$

$$U = \sqrt{\frac{Y}{\rho}} \quad ----(5)$$

$$U = K + \frac{4}{3}\eta$$

$$U = \sqrt{\frac{K + \frac{4}{3}\eta}{\rho}}$$

5.13: Wave Velocity and Molecular Velocity:

$$U = \sqrt{\frac{\gamma P}{\rho}} \quad ----(1)$$

According to this equation the velocity of sound will depend on the pressure of the gas. Also, according to kinetic theory of gases, the pressure of the gas is;

$$P = \frac{1}{3}\rho c^2$$

$$\frac{P}{\rho} = \frac{c^2}{3} \quad ----(2)$$

$$U = \sqrt{\frac{\gamma c^2}{3}} \quad ----(3)$$

$$or \quad U = c\sqrt{\frac{\gamma}{3}} \quad ----(3)$$

$$c = U\sqrt{\frac{3}{\gamma}} \quad ----(4)$$

From equation (3), if the value of velocity of sound and the ratio of the two specific heats of the gas are known, the root mean square velocity of the molecules can be calculated.