Chapter One- Part (II)
Vector Analysis and Coordinate Systems

1-1: Introduction:

The use of vectors and vector analysis can greatly simplify the mathematics used in expressing and manipulating the laws and theorems of electric and magnetic fields.

1-2: Scalars and Vectors:

Scalar is a quantity that has only magnitude, such as: time, distance, temperature, entropy, energy, mass, electric potential ....

The vector is a quantity that has both magnitude and direction, such as: velocity, force, displacement, electric and magnetic field intensity ....

The vector $\vec{A}$ is denoted in Cartesian coordinate as:

$$\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z \quad or \quad \vec{A} = A\hat{a}_a$$

$$where,\ \hat{a}_a\ is\ a\ unit\ vector$$

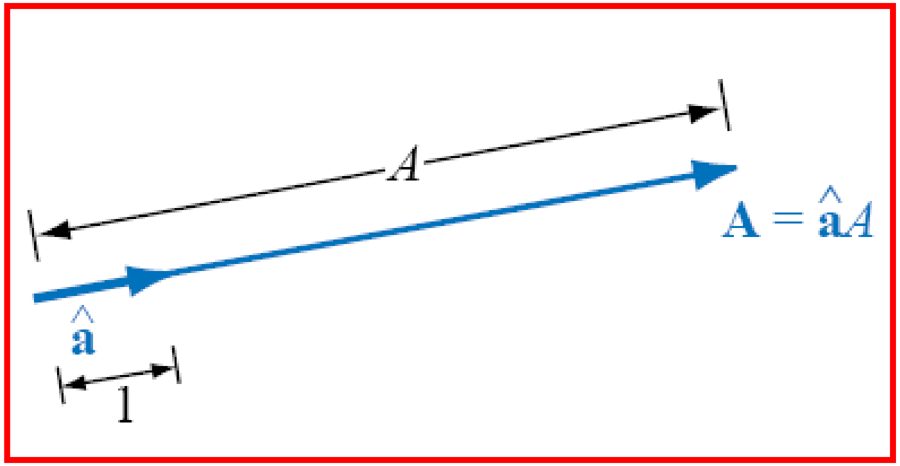
1-3: Unit Vectors:

The unit vector of $\vec{A}$ is defined as a vector whose magnitude is unity and its direction is along $\vec{A}$, that is:

$$\hat{a}_a = \frac{\vec{A}}{|A|} \quad ,$$

$$\vec{A} = A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z$$

$$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$



Where, $A_x, A_y \; and \; A_z$ are component of $\vec{A}$ in the $x, y \; and \; z$ direction respectively.

1-4: Equality of Two Vectors:

The vectors $\vec{A} \; and \; \vec{B}$ are said to be equal if they have equal magnitudes and identical unit vectors. Thus, if:

$$\vec{A} = A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z \quad or \quad \vec{A} = A\hat{a}_a$$

$$\vec{B} = B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z \quad or \quad \vec{B} = B\hat{a}_b$$

Then $\vec{A} = \vec{B}$ if and only if $|A| = |B| \quad and \quad \hat{a}_a = \hat{a}_b$

1. When two parallel vectors have the same magnitude and pointing to the same direction are equal.

2. but they are identical only if the lie on top of one another

1-5: Vector addition and subtraction:

Two vectors $\vec{A} \ and \ \vec{B}$ can be added together to give another vector $\vec{C}$ that is:

$$\vec{C} = \vec{A} + \vec{B}$$

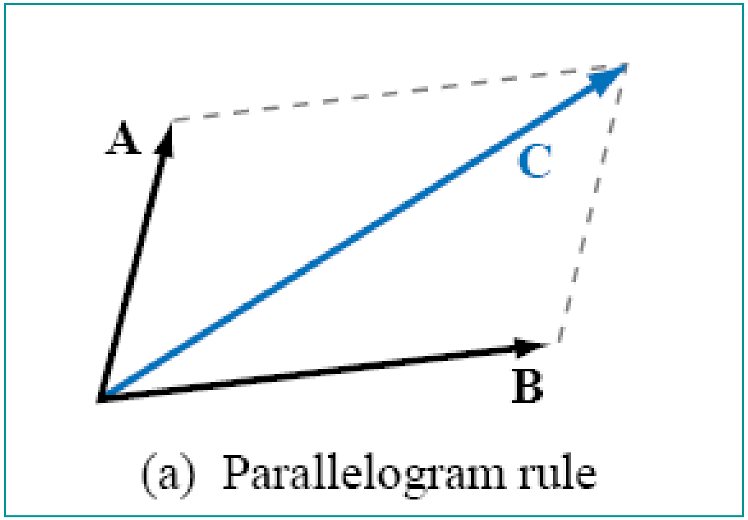
$$\vec{C} = (A_x + B_x)\hat{a}_x + (A_y + B_y)\hat{a}_y + (A_z + B_z)\hat{a}_z$$

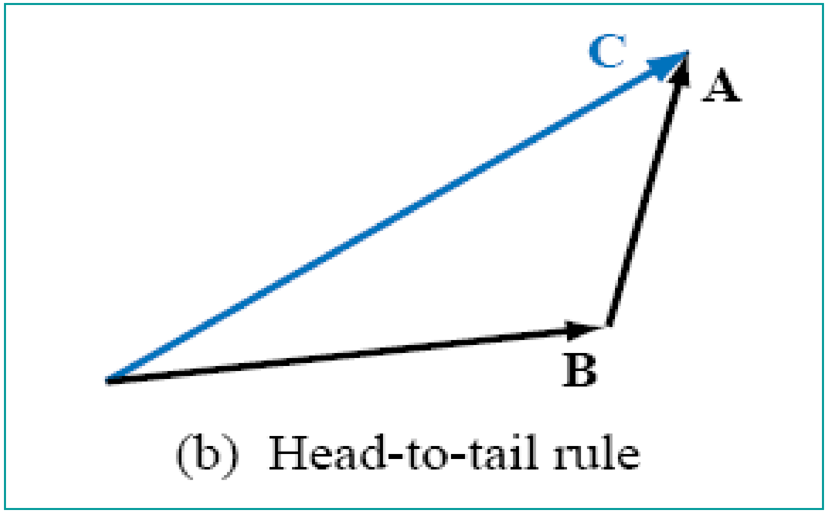
$$\vec{C} = C_x\hat{a}_x + C_y\hat{a}_y + C_z\hat{a}_z$$

And vector subtraction is similarly carried out as:

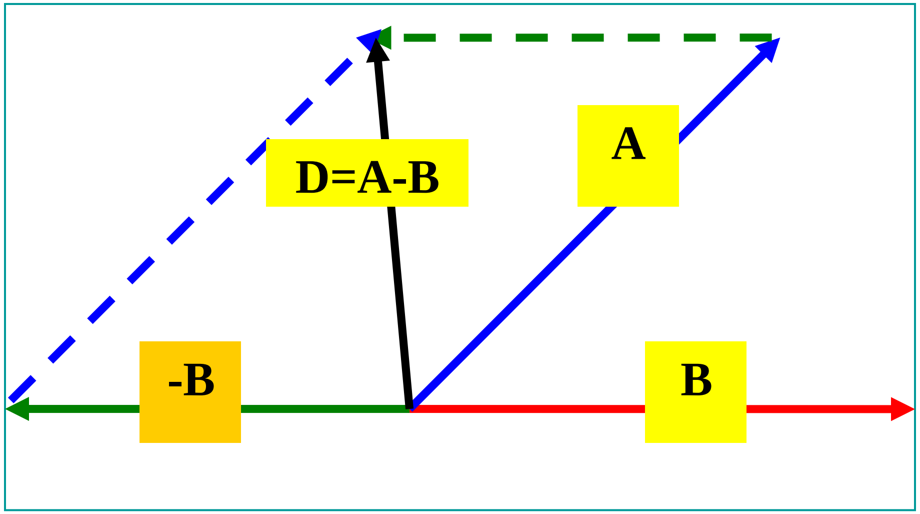
$$\vec{D} = \vec{A} - \vec{B} \Rightarrow \quad \vec{D} = (A_x - B_x)\hat{a}_x + (A_y - B_y)\hat{a}_y + (A_z - B_z)\hat{a}_z$$

$$\vec{D} = D_x\hat{a}_x + D_y\hat{a}_y + D_z\hat{a}_z$$





No.	Law	Addition	Multiplication
1.	**Commutative**	$\vec{A}+\vec{B}=\vec{B}+\vec{A}$	$k\ \vec{A}=\vec{A}k$
2.	**Associative**	$\vec{A}+(\vec{B}+\vec{C})=(\vec{A}+\vec{B})+\vec{C}$	$k(l\vec{A})=l(k\vec{A})$
3.	**Distributive**	$k(\vec{A}+\vec{B})=k\vec{A}+k\vec{B}$	-----------



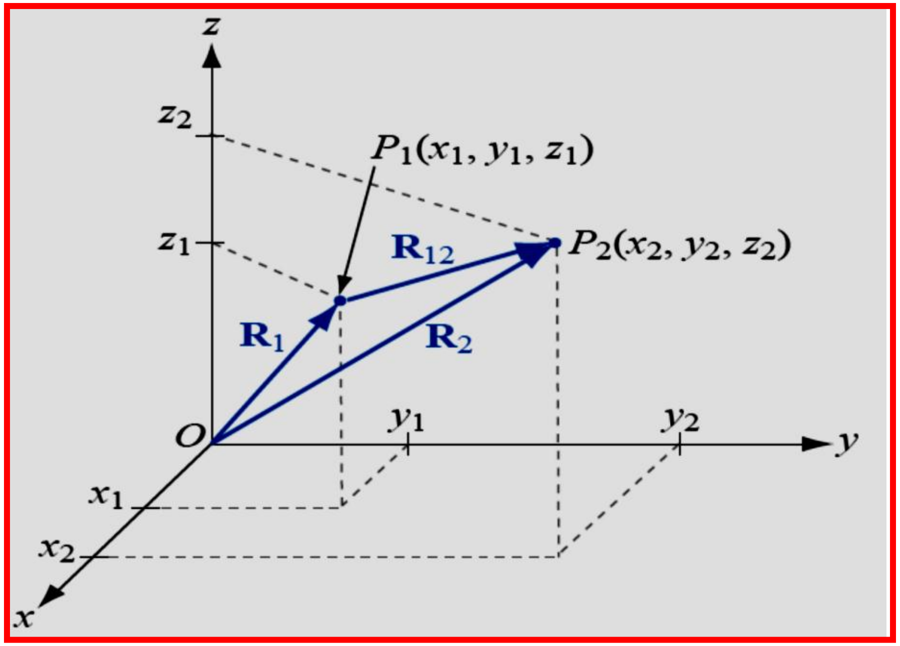
1-6: Position and Distance Vector:

The position vector $\vec{R}_p$ or radius vector of point $P(x,y,z)$ is defined as the directed distance from the origin $O(0,0,0)$ to point $P(x,y,z)$ as: $\vec{R}_p = \overline{OP} = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z$

The distance vector is the displacement from one point to another, such as, $P_1(x_1, y_1, z_1)$ to $P_2(x_2, y_2, z_2)$ and is represented mathematically as:

$$\vec{R}_{12} = \overline{P_1P_2} = (x_2 - x_1)\,\hat{a}_x + (y_2 - y_1)\hat{a}_y + (z_2 - z_1)\,\hat{a}_z$$

Therefore, the position vector is a special case of displacement vector.



Example(1): Given points $P(1,-3,5)$, $Q(2,4,6)$ *and* $R(0,3,8)$ then find the following:

(a). the position vectors of P and R (b). the distance vector $\vec{\mathbf{r}}_{QP}$ (c). the distance between Q and R

Solution:

(a). $\overline{OP} = (1-0)\hat{a}_x + (-3-0)\hat{a}_y + (5-0)\hat{a}_z = \hat{a}_x + -3\hat{a}_y + 5\hat{a}_z$

$\overline{OR} = (0-0)\hat{a}_x + (3-0)\hat{a}_y + (8-0)\hat{a}_z = 3\hat{a}_y + 8\hat{a}_z$

(b). $\vec{\mathbf{r}}_{QP} = (1-2)\hat{a}_x + (-3-4)\hat{a}_y + (5-6)\hat{a}_z = -\hat{a}_x - 7\hat{a}_y - \hat{a}_z$

(c). $\left|\overline{QR}\right| = \sqrt{(0-2)^2 + (3-4)^2 + (8-6)^2} = \sqrt{4+1+4} = 3\,unit$

1-7: Vector Multiplication:

Three types of products can occur in vector calculus. These are, simple, scalar(or dot) and vector(or cross) products:

1-7-1: Simple Product:

Multiplication of a vector by a scalar quantity is called a simple product and is represented mathematically as:

$$\vec{A} = A\hat{a}_a$$

$$k\,\vec{A} = k\,A\hat{a}_a = \vec{B}$$

$$\vec{B} = k\,A_x\,\hat{a}_x + k\,A_y\,\hat{a}_y + k\,A_z\,\hat{a}_z$$

$$\vec{B} = B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z$$

$\vec{B}$ *is in the same direction of* $\vec{A}$

Such as $\vec{F}_e = Q\,\vec{E}$

1-7-2: Scalar Product:

The scalar or dot product of two vectors yields a scalar quantity and is denoted by:

$$\vec{A}\cdot\vec{B} = |A|\,|B|\,\cos\theta_{AB}$$

$$W = \int \vec{F}\cdot\overline{dl}$$

And obey the following algebraic relation:

$$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$$

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

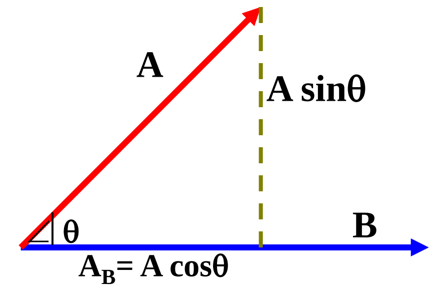
$$\vec{A}\cdot\vec{A} = |A|^2 = A^2 = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\hat{a}_x\cdot\hat{a}_x = \hat{a}_y\cdot\hat{a}_y = \hat{a}_z\cdot\hat{a}_z = 1 \quad \theta_{AB} = 0$$

$$\hat{a}_x\cdot\hat{a}_y = \hat{a}_y\cdot\hat{a}_z = \hat{a}_z\cdot\hat{a}_x = 0 \quad \theta_{AB} = 90$$

The scalar component of vector $\vec{A}\ along\ \vec{B}$ is represented mathematically as:

$$A_B = |A|\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|B|} = \vec{A}\cdot\hat{a}_B$$



and the vector component of $\vec{A}\ along\ \vec{B}$ is represented mathematically as:

$$\vec{A}_B = |A|\cos\theta\ \hat{a}_B = \frac{\vec{A}\cdot\vec{B}}{|B|}\ \hat{a}_B = (\vec{A}\cdot\hat{a}_B)\,\hat{a}_B$$

While the normal component of $\vec{A}\ on\ \vec{B}$ can be determined as: $\vec{A}_{NB} = \vec{A} - \vec{A}_B$

In addition the scalar projection of vector $\vec{A}$ onto $z-axis$ is given by:

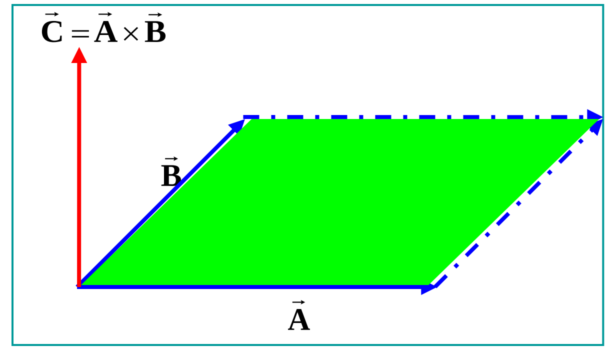
$$A_z = \frac{\hat{a}_z\cdot\vec{A}}{|\hat{a}_z|} = \hat{a}_z\cdot\vec{A}$$

1-7-3: Vector Product:

The vector or cross product of two vectors such as $\vec{A}$ *and* $\vec{B}$ yields another vector which perpendicular to both $\vec{A}$ *and* $\vec{B}$ and is pronounced $\vec{A}$ *cross* $\vec{B}$ and is denoted by:

$$\vec{A}\times\vec{B} = |A|\,|B|\sin\theta = \vec{C} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_yB_z - A_zB_y)\hat{a}_x + (A_zB_x - A_xB_z)\hat{a}_y + (A_xB_y - A_yB_x)\hat{a}_z$$

Where $\vec{C}$ is perpendicular to both $\vec{A}$ *and* $\vec{B}$ .The vector products obey the following algebraic relation:



$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$$

$$\vec{A}\times(\vec{B}\times\vec{C}) \neq (\vec{A}\times\vec{B})\times\vec{C}$$

$$\vec{A}\times\vec{A} = zero$$

$$\vec{A}\times(\vec{B}\pm\vec{C}) = \vec{A}\times\vec{B}\pm\vec{A}\times\vec{C}$$

$$\hat{a}_x\times\hat{a}_x = \hat{a}_y\times\hat{a}_y = \hat{a}_z\times\hat{a}_z = zero$$

$$\hat{a}_x\times\hat{a}_y = \hat{a}_z \qquad \hat{a}_y\times\hat{a}_z = \hat{a}_x \qquad \hat{a}_z\times\hat{a}_x = \hat{a}_y$$

$$\hat{a}_y\times\hat{a}_x = -\hat{a}_z \qquad \hat{a}_z\times\hat{a}_y = -\hat{a}_x \qquad \hat{a}_x\times\hat{a}_z = -\hat{a}_y$$

$$\vec{\tau} = \vec{F}\times\vec{r}$$

$$\vec{F} = q\vec{v}\times\vec{B}$$

The magnitude of $\vec{A}\times\vec{B}$ is the area of the parallelogram formed by $\vec{A}$ *and* $\vec{B}$ as shown in the figure. Therefore, the area of triangular formed by $\vec{A}$ *and* $\vec{B}$ is . $\left|\frac{1}{2}\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right|$

Example(2): **Given vectors** $\vec{\mathbf{A}} = \hat{a}_x + 3\hat{a}_z \quad and \quad \vec{\mathbf{B}} = 5\hat{a}_x + 2\hat{a}_y - 6\hat{a}_z$ **, then determine the following:**

(a). $|\mathbf{A}+\mathbf{B}|$ **(c). the component of** $\mathbf{A}\ along\ \hat{a}_y$

(b). $3\vec{\mathbf{A}} - \vec{\mathbf{B}}$ **(d). the unit vector parallel to** $3\vec{\mathbf{A}} + \vec{\mathbf{B}}$

Solution:

(a). $\vec{\mathbf{A}}+\vec{\mathbf{B}} = (\hat{a}_x + 3\hat{a}_z) + (5\hat{a}_x + 2\hat{a}_y - 6\hat{a}_z) = 6\hat{a}_x + 2\,\hat{a}_y - 3\,\hat{a}_z$

$$\left|\vec{\mathbf{A}}+\vec{\mathbf{B}}\right| = \sqrt{36+4+9} = 7\,unit$$

(b). $3\vec{\mathbf{A}}-\vec{\mathbf{B}} = (3\hat{a}_x + 9\hat{a}_z) - (5\hat{a}_x + 2\hat{a}_y - 6\hat{a}_z) = -2\hat{a}_x - 2\hat{a}_y + 15\hat{a}_z$

(c). the scalar component of $\mathbf{A}\ along\ \hat{a}_y$ **is given by:**

$$A_y = \vec{A}\cdot\hat{a}_y = \frac{\vec{A}\cdot\hat{a}_y}{|\hat{a}_y|} = \frac{(\hat{a}_x + 3\hat{a}_z)\cdot\hat{a}_y}{|\hat{a}_y|} = \frac{0}{1} = 0$$

The vector component of $\mathbf{A}\ along\ \hat{a}_y$ **is given by:**

$$\vec{A}_y = (\vec{A}\cdot\hat{a}_y)\hat{a}_y = \frac{\vec{A}\cdot\hat{a}_y}{|\hat{a}_y|}\,\hat{a}_y = (0)\hat{a}_y = 0$$

(d). $3\vec{\mathbf{A}}+\vec{\mathbf{B}} = (3\hat{a}_x + 9\hat{a}_z) + (5\hat{a}_x + 2\hat{a}_y - 6\hat{a}_z) = 8\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z$ **, then the unit vector parallel to this vector is calculated as:**

$$\hat{a}_{3\vec{\mathbf{A}}+\vec{\mathbf{B}}} = \frac{3\vec{\mathbf{A}}+\vec{\mathbf{B}}}{\left|3\vec{\mathbf{A}}+\vec{\mathbf{B}}\right|} = \frac{8\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z}{\sqrt{64+4+9}} = \frac{8\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z}{\sqrt{77}}$$

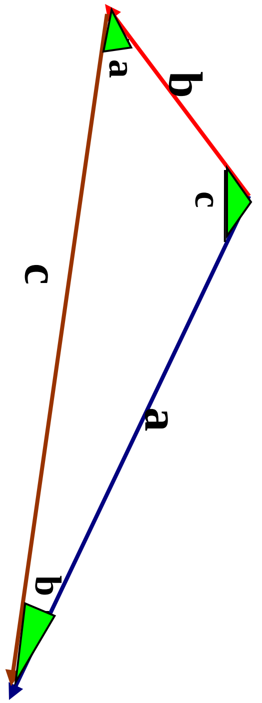
Example(3): Derive the cosine formula $c^2 = a^2 + b^2 + 2ab\cos\theta$ **and the sine formula** $\dfrac{\sin a}{a} = \dfrac{\sin b}{b} = \dfrac{\sin c}{c}$ **using dot and cross product respectively.**

Cosine formula:

$$\vec{\mathbf{c}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$$

$$\vec{\mathbf{c}} \cdot \vec{\mathbf{c}} = (\vec{\mathbf{a}} + \vec{\mathbf{b}}) \cdot (\vec{\mathbf{a}} + \vec{\mathbf{b}}) = \mathbf{a}^2 + \mathbf{b}^2 + 2\,\mathbf{a} \cdot \mathbf{b}$$

$$\mathbf{c}^2 = \mathbf{a}^2 + \mathbf{b}^2 + 2\,\mathbf{ab}\cos\vartheta \qquad \Rightarrow \quad \mathbf{c} = \sqrt{\mathbf{a}^2 + \mathbf{b}^2 + 2\,\mathbf{ab}\cos\vartheta}$$



Sine formula:

$$\left|\frac{1}{2}\vec{\mathbf{a}} \times \vec{\mathbf{b}}\right| = \left|\frac{1}{2}\vec{\mathbf{b}} \times \vec{\mathbf{c}}\right| = \left|\frac{1}{2}\vec{\mathbf{c}} \times \vec{\mathbf{a}}\right|$$

$$\therefore \frac{1}{2}\,\mathbf{a}\,\mathbf{b}\sin\mathbf{c} = \frac{1}{2}\,\mathbf{b}\,\mathbf{c}\,\sin\mathbf{a} = \frac{1}{2}\,\mathbf{c}\,\mathbf{a}\,\sin\mathbf{b}$$

dividing the above equation by $(\mathbf{a}\,\mathbf{b}\,\mathbf{c})$ *we get :*

$$\frac{\sin\mathbf{c}}{\mathbf{c}} = \frac{\sin\mathbf{a}}{\mathbf{a}} = \frac{\sin\mathbf{b}}{\mathbf{b}}$$

Example(4): Let $\vec{\mathbf{E}} = 3\hat{a}_y + 4\hat{a}_z \quad and \quad \vec{\mathbf{F}} = 4\hat{a}_x - 10\hat{a}_y + 5\hat{a}_z$, then find:

(a). the component of $\mathbf{E}$ *along* $\mathbf{F}$ (b). A unit vector perpendicular to both $\mathbf{E}$ *and* $\mathbf{F}$

Solution:

(a). the vector component of $\mathbf{E}$ *along* $\mathbf{F}$ is given by:

$$\vec{\mathbf{E}}_{\mathbf{F}} = |\mathbf{E}|\cos\theta\ \hat{a}_{\mathbf{F}} = \frac{\vec{\mathbf{E}}\cdot\vec{\mathbf{F}}}{|\mathbf{F}|}\ \hat{a}_{\mathbf{F}} = (\vec{\mathbf{E}}\cdot\hat{a}_{\mathbf{F}})\,\hat{a}_{\mathbf{F}}$$

$$\hat{a}_{\mathbf{F}} = \frac{4\hat{a}_x - 10\hat{a}_y + 5\hat{a}_z}{\sqrt{16+100+25}} = \frac{4\hat{a}_x - 10\hat{a}_y + 5\hat{a}_z}{\sqrt{141}}$$

$$\therefore\ (\vec{\mathbf{E}}\cdot\hat{a}_{\mathbf{F}})\,\hat{a}_{\mathbf{F}} = \left[(3\hat{a}_x + 4\hat{a}_z)\cdot\frac{4\hat{a}_x - 10\hat{a}_y + 5\hat{a}_z}{\sqrt{141}}\right]\frac{4\hat{a}_x - 10\hat{a}_y + 5\hat{a}_z}{\sqrt{141}}$$

$$\therefore\ \vec{\mathbf{E}}_{\mathbf{F}} = \frac{(12+20)}{141}\,(4\hat{a}_x - 10\hat{a}_y + 5\hat{a}_z)$$

The scalar component of $\mathbf{E}$ *along* $\mathbf{F}$ is given by:

$$\vec{\mathbf{E}}_{\mathbf{F}} = |\mathbf{E}|\cos\theta = \frac{\vec{\mathbf{E}}\cdot\mathbf{F}}{|\mathbf{F}|} = \vec{\mathbf{E}}\cdot\hat{a}_{\mathbf{F}}$$

$$\vec{\mathbf{E}}_{\mathbf{F}} = \vec{\mathbf{E}}\cdot\hat{a}_{\mathbf{F}} = \left[(3\hat{a}_x + 4\hat{a}_z)\cdot\frac{4\hat{a}_x - 10\hat{a}_y + 5\hat{a}_z}{\sqrt{141}}\right] = \frac{12+20}{\sqrt{141}}$$

1-8: Scalar and Vector Triple Product:

When three vectors are multiplied, not all combinations of dot or cross products are meaningful. For example the product: $\vec{A}\times(\vec{B}\cdot\vec{C}) \quad , \; (\vec{A}\times\vec{B})\vec{C}$ not defined under the rules of algebra.

Therefore, the only two meaningful products of three vectors are the Scalar triple and the Vector product as represented them below:

1-8-1: Scalar Triple Product:

A scalar triple product denoted $\vec{\mathbf{A}}\cdot(\vec{\mathbf{B}}\times\vec{\mathbf{C}})$ and obeying the following cyclic order:

$$\vec{\mathbf{A}}\cdot(\vec{\mathbf{B}}\times\vec{\mathbf{C}})=\vec{\mathbf{C}}\cdot(\vec{\mathbf{A}}\times\vec{\mathbf{B}})=\vec{\mathbf{B}}\cdot(\vec{\mathbf{C}}\times\vec{\mathbf{A}})$$

where,
$$\vec{\mathbf{A}}\cdot(\vec{\mathbf{B}}\times\vec{\mathbf{C}})=\begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

1-8-2: Vector Triple Product:

The vector triple product involves the cross product of a vector with the cross product of two others, such as:

$$\vec{\mathbf{A}}\times(\vec{\mathbf{B}}\times\vec{\mathbf{C}})\neq(\vec{\mathbf{A}}\times\vec{\mathbf{B}})\times\vec{\mathbf{C}}$$

$$\vec{\mathbf{A}}\times(\vec{\mathbf{B}}\times\vec{\mathbf{C}})=\vec{\mathbf{B}}(\vec{\mathbf{A}}\cdot\vec{\mathbf{C}})-\vec{\mathbf{C}}(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}})$$ very important rule----prove this (H. W.)

Example(5): Three field quantities are given by: $\vec{\mathbf{P}} = 2\hat{a}_x - \hat{a}_z$, $\vec{\mathbf{Q}} = 2\hat{a}_x - \hat{a}_y + 2\hat{a}_z$, $\vec{\mathbf{R}} = 2\hat{a}_x - 3\hat{a}_y + \hat{a}_z$ **determine the following:**

(a). $(\vec{\mathbf{P}}+\vec{\mathbf{Q}})\times(\vec{\mathbf{P}}-\vec{\mathbf{Q}}) = \vec{\mathbf{P}}\times(\vec{\mathbf{P}}-\vec{\mathbf{Q}}) + \vec{\mathbf{Q}}\times(\vec{\mathbf{P}}-\vec{\mathbf{Q}})$ **(b).** $\vec{\mathbf{Q}}\cdot(\vec{\mathbf{R}}\times\vec{\mathbf{P}})$ **(c).** $\sin\theta_{QR}$

(d). $\vec{\mathbf{P}}\times(\vec{\mathbf{Q}}\times\vec{\mathbf{R}})$ **(e). A unit vector perpendicular to both** $\vec{\mathbf{Q}}$ *and* $\vec{\mathbf{R}}$

Solution:

(a).

$$(\vec{\mathbf{P}}+\vec{\mathbf{Q}})\times(\vec{\mathbf{P}}-\vec{\mathbf{Q}}) = \vec{\mathbf{P}}\times(\vec{\mathbf{P}}-\vec{\mathbf{Q}}) + \vec{\mathbf{Q}}\times(\vec{\mathbf{P}}-\vec{\mathbf{Q}}) = \vec{\mathbf{P}}\times\vec{\mathbf{P}} - \vec{\mathbf{P}}\times\vec{\mathbf{Q}} + \vec{\mathbf{Q}}\times\vec{\mathbf{P}} - \vec{\mathbf{Q}}\times\vec{\mathbf{Q}}$$

$$= 0 + \vec{\mathbf{Q}}\times\vec{\mathbf{P}} + \vec{\mathbf{Q}}\times\vec{\mathbf{P}} - 0 = 2\left|\vec{\mathbf{P}}\right|\left|\vec{\mathbf{Q}}\right| \sin\theta_{QP}$$

$$\left|\vec{\mathbf{P}}\right| = \sqrt{4+1} = \sqrt{5} \qquad \left|\vec{\mathbf{Q}}\right| = \sqrt{4+1+4} = 3$$

$$\theta_{QP} = \cos^{-1}\frac{\vec{\mathbf{Q}}\cdot\vec{\mathbf{P}}}{\left|\vec{\mathbf{P}}\right|\left|\vec{\mathbf{Q}}\right|} = \frac{4-2}{3\times\sqrt{5}} = \frac{2}{3\sqrt{5}} = 72.65^\circ$$

$$\sin 72.65 = 0.95$$

(b).

$$\vec{\mathbf{Q}}\cdot(\vec{\mathbf{R}}\times\vec{\mathbf{P}}) = \begin{vmatrix} 2 & -1 & 2 \\ 2 & -3 & 1 \\ 2 & 0 & -1 \end{vmatrix} = 2\times(3-0) - 1\times(2+2) + 2\times(0+6) = 6-4+12 = 14$$

(c).

$$\sin\theta_{QR} = \frac{\left|\vec{\mathbf{Q}}\times\vec{\mathbf{R}}\right|}{\left|\vec{\mathbf{Q}}\right|\left|\vec{\mathbf{R}}\right|} \qquad \vec{\mathbf{Q}}\times\vec{\mathbf{R}} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 2 & -1 & 2 \\ 2 & -3 & 1 \end{vmatrix} = \hat{a}_x(-1+6) + \hat{a}_y(4-2) + \hat{a}_z(-6+2) = 5\hat{a}_x + 2\hat{a}_y - 4\hat{a}_z$$

$$\left|\vec{\mathbf{Q}}\times\vec{\mathbf{R}}\right| = \sqrt{25+4+16} = \sqrt{45} \quad , \quad \left|\vec{\mathbf{Q}}\right| = \sqrt{4+1+4} = 3 \quad \textit{and} \quad \left|\vec{\mathbf{R}}\right| = \sqrt{4+9+1} = \sqrt{14}$$

$$\sin\theta_{QR} = \frac{\sqrt{45}}{3\sqrt{14}} = 0.596$$

(d). $\vec{\mathbf{P}}\times(\vec{\mathbf{Q}}\times\vec{\mathbf{R}}) = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 2 & 0 & -1 \\ 5 & 2 & -4 \end{vmatrix} = \hat{a}_x(0+2)+\hat{a}_y(-5+8)+\hat{a}_z(4-0) = 2\hat{a}_x + 3\hat{a}_y + 4\hat{a}_z$

(e). $\hat{a}_{QR} = \dfrac{\vec{\mathbf{Q}}\times\vec{\mathbf{R}}}{|\vec{\mathbf{Q}}||\vec{\mathbf{R}}|} = \dfrac{5\hat{a}_x + 2\hat{a}_y - 4\hat{a}_z}{3\sqrt{14}}$

(f). The scalar and vector component of $\vec{\mathbf{P}}$ *along* $\vec{\mathbf{Q}}$ **can be calculated as follows:**

The scalar component of $\vec{\mathbf{P}}$ *along* $\vec{\mathbf{Q}}$ **is equal to:**

$$\vec{\mathbf{P}}_{\mathbf{Q}} = |\mathbf{P}|\cos\theta = \frac{\vec{\mathbf{P}}\cdot\vec{\mathbf{Q}}}{|\mathbf{Q}|} = \vec{\mathbf{P}}\cdot\hat{a}_{\mathbf{Q}}$$

$$\vec{\mathbf{P}}\cdot\hat{a}_{\mathbf{Q}} = \left[(2\hat{a}_x - \hat{a}_z)\cdot\frac{2\hat{a}_x - \hat{a}_y + 2\hat{a}_z}{3}\right] = \frac{4-2}{3} = \frac{2}{3}$$

The vector component of $\vec{\mathbf{P}}$ *along* $\vec{\mathbf{Q}}$ **is given by:**

$$\vec{\mathbf{P}}_{\mathbf{Q}} = |\mathbf{E}|\cos\theta\ \hat{a}_{\mathbf{F}} = \frac{\vec{\mathbf{P}}\cdot\vec{\mathbf{Q}}}{|\vec{\mathbf{Q}}|}\ \hat{a}_{\mathbf{Q}} = (\vec{\mathbf{P}}\cdot\hat{a}_{\mathbf{Q}})\hat{a}_{\mathbf{Q}}$$

$$\hat{a}_{\mathbf{Q}} = \frac{2\hat{a}_x - \hat{a}_y + 2\hat{a}_z}{3}$$

$$\therefore\ (\vec{\mathbf{P}}\cdot\hat{a}_{\mathbf{Q}})\hat{a}_{\mathbf{Q}} = \left[(2\hat{a}_x - \hat{a}_z)\cdot\frac{2\hat{a}_x - \hat{a}_y + 2\hat{a}_z}{3}\right]\frac{2\hat{a}_x - \hat{a}_y + 2\hat{a}_z}{3}$$

$$\therefore\ \vec{\mathbf{P}}_{\mathbf{Q}} = \frac{4-2}{9}(2\hat{a}_x - \hat{a}_y + 2\hat{a}_z) = \frac{2}{9}(2\hat{a}_x - \hat{a}_y + 2\hat{a}_z)$$

Home Work

Q1/ Given $\vec{A}=\hat{a}_x-\hat{a}_y\ ,\quad \vec{B}=2\hat{a}_z\quad and\quad \vec{C}=-\hat{a}_x+3\hat{a}_y$ **, find both** $\vec{A}\cdot\vec{B}\times\vec{C}\quad and\quad (\vec{A}\times\vec{B})\times\vec{C}$

Q2 / Show that : .

$$(\vec{A}\cdot\vec{B})^2+(\vec{A}\times\vec{B})^2=(AB)^2$$

Q3 / Find the triple scalar and vector product of the following vectors :

$$\vec{P}=2\hat{a}_x-2\hat{a}_y+\hat{a}_z\quad ,\ \vec{Q}=\hat{a}_x+\hat{a}_y+\hat{a}_z\quad and\quad \vec{R}=2\hat{a}_x+3\hat{a}_z$$

Q4/ Find the angle between $\vec{A}=10\hat{a}_y+2\hat{a}_z\quad ,\quad and\quad \vec{B}=-4\hat{a}_y+0.5\hat{a}_z$

using both dot and cross product.

Q5/ Given $\vec{A}=(y-1)\hat{a}_x+2x\hat{a}_y$**, find the vector at point** $(2,2,1)$ **and its projection on vector** $\vec{B}$ **where** $\vec{B}=5\hat{a}_x-\hat{a}_y+2\hat{a}_z$

Q6/ Given vectors $\vec{A} = \hat{a}_x + 2\hat{a}_y - 3\hat{a}_z$, $\vec{B} = 3\hat{a}_x - 4\hat{a}_y$ *and* $\vec{C} = 3\hat{a}_y - 4\hat{a}_z$

find the following :

a. $|\vec{A}|$ *and* $\hat{a}_A$

b. the component of $\vec{B}$ *along* $\vec{C}$

c. θ_{AC}

d. $\vec{A} \times \vec{C}$

e. $\vec{A} \times (\vec{B} \times \vec{C})$

f. $\hat{a}_x \times \vec{B}$

g. $(\vec{A} \times \hat{a}_y).\hat{a}_z$

h. $\vec{A} \cdot (\vec{B} \times \vec{C})$

Q7/ Express the unit vector directed toward the point $(0,0,h)$ from an arbitrary point in the plane $z = -2$. Explain the result as $h = -2$?

Q8/ Given $\vec{A} = 4\hat{a}_y + 10\hat{a}_z$ *and* $\vec{B} = 2\hat{a}_x + 3\hat{a}_y$ **, find the magnitude and vector components of** $\vec{A}$ *on* $\vec{B}$ **?**

Q9 /Given Vectors $\vec{T} = 2\hat{a}_x - 6\hat{a}_y + 3\hat{a}_z$ *and* $\vec{S} = \hat{a}_x + 2\hat{a}_y + \hat{a}_z$ **then find the following :**

(a)- The scalar projection of $\vec{T}$ *on* $\vec{S}$

(b)-The vector projection of $\vec{S}$ *on* $\vec{T}$

(c)- The angle between $\vec{T}$ *and* $\vec{S}$