2-3-3: Curl of a Vector and Stokes's Theorem

Stokes's theorem, states that the circulation of a vector field ($\vec{\mathbf{A}}$) around a closed path (L) is equal to the surface integral of the curl of ($\vec{\mathbf{A}}$) over the open surface (S)bounded by (L) as given in the following figure, provided that ($\vec{\mathbf{A}}$) and Curl continuous on S.

$$\oint_L \vec{\mathbf{A}} \cdot \overrightarrow{\mathbf{dl}} = \oint_L \vec{\nabla} \times \vec{\mathbf{A}} \cdot \overrightarrow{\mathbf{ds}} \quad ------------------(1)$$

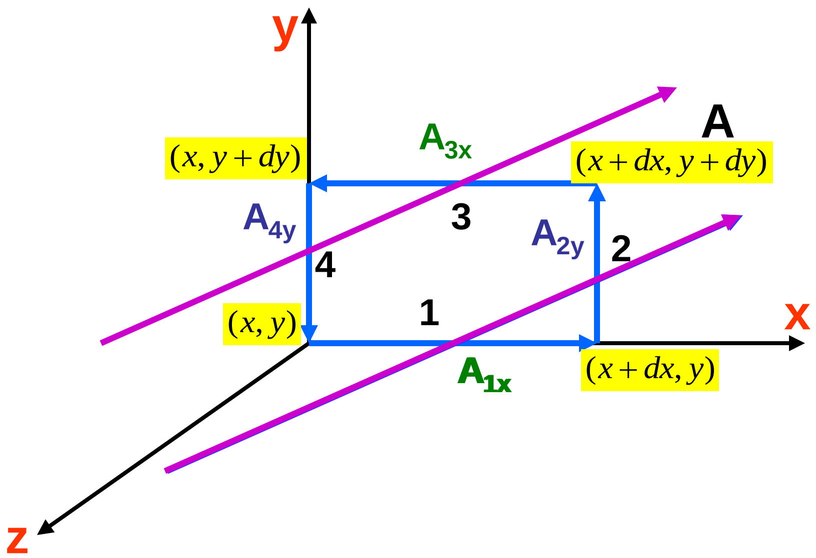
From this figure the line integral around each path is given as:

The line integral around path (1)= $\mathbf{A}_{1x}\,dx$

The line integral around path (2)= $\mathbf{A}_{2y}\,dy$

The line integral around path (3)= $-\ \mathbf{A}_{3x}\,dx$

The line integral around path (4)= $-\ \mathbf{A}_{4y}\,dy$



$$\oint_L \vec{\mathbf{A}} \cdot \overrightarrow{\mathbf{dl}} = \oint_1 \vec{\mathbf{A}} \cdot \overrightarrow{\mathbf{dl}} + \oint_2 \vec{\mathbf{A}} \cdot \overrightarrow{\mathbf{dl}} + \oint_3 \vec{\mathbf{A}} \cdot \overrightarrow{\mathbf{dl}} + \oint_4 \vec{\mathbf{A}} \cdot \overrightarrow{\mathbf{dl}} \quad ------------------(2)$$

Hence, $$\oint_L \vec{\mathbf{A}}\cdot\overrightarrow{\mathbf{dl}} = (\mathbf{A}_{1x} - \mathbf{A}_{3x})\,dx + (\mathbf{A}_{2y} - \mathbf{A}_{4y})\,dy \text{ ----------- } (3)$$

According to Taylor series we have:

$$\left.\begin{aligned} \mathbf{A}_{3x} &= \mathbf{A}_{1x} + \frac{\partial \mathbf{A}_x}{\partial y}\,dy \\ \mathbf{A}_{2y} &= \mathbf{A}_{4y} + \frac{\partial \mathbf{A}y}{\partial x}\,dx \end{aligned}\right\} \text{ ----- } (4)$$

Substituting eq.(4) into eq.(3) we get:

$$\oint_L \vec{\mathbf{A}}\cdot\overrightarrow{\mathbf{dl}} = (\mathbf{A}_{1x} - \mathbf{A}_{1x} - \frac{\partial \mathbf{A}_x}{\partial y}\,dy)\,dx + (\mathbf{A}_{4y} + \frac{\partial \mathbf{A}_y}{\partial x}\,dx - \mathbf{A}_{4y})\,dy = -\frac{\partial \mathbf{A}_x}{\partial y}\,dy\,dx + \frac{\partial \mathbf{A}_y}{\partial x}\,dx\,dy$$

$$\oint_L \vec{\mathbf{A}}\cdot\overrightarrow{\mathbf{dl}} = -\frac{\partial \mathbf{A}_x}{\partial y}\,dx\,dy + \frac{\partial \mathbf{A}_y}{\partial x}\,dx\,dy == (curl\ \mathbf{A})\Big|_{z-plane} = \left(\frac{\partial \mathbf{A}_y}{\partial x} - \frac{\partial \mathbf{A}_x}{\partial y}\right)dx\,dy \text{ ----------- } (5)$$

By the same way we can evaluate the line integrals around a closed path in both y and x-planes as given below:

$$\left.\begin{aligned} \oint_L \vec{\mathbf{A}}\cdot\overrightarrow{\mathbf{dl}} &= (curl\ \mathbf{A})\Big|_{y-plane} = \left(\frac{\partial \mathbf{A}_x}{\partial z} - \frac{\partial \mathbf{A}_z}{\partial x}\right)dx\,dz \\ \oint_L \vec{\mathbf{A}}\cdot\overrightarrow{\mathbf{dl}} &= (curl\ \mathbf{A})\Big|_{x-plane} = \left(\frac{\partial \mathbf{A}_z}{\partial y} - \frac{\partial \mathbf{A}_y}{\partial z}\right)dy\,dz \end{aligned}\right\} \text{ ------------- } (6)$$

Adding equations (5) and (6) we get:

$$\oint_L \vec{\mathbf{A}} \cdot \overrightarrow{\mathbf{dl}} = \left(\frac{\partial \mathbf{A}_y}{\partial x} - \frac{\partial \mathbf{A}_x}{\partial y}\right) dx\,dy + \left(\frac{\partial \mathbf{A}_x}{\partial z} - \frac{\partial \mathbf{A}_z}{\partial x}\right) dx\,dz + \left(\frac{\partial \mathbf{A}_z}{\partial y} - \frac{\partial \mathbf{A}_y}{\partial z}\right) dz\,dy - - - - - - - -(7)$$

Since, the circulation of a vector (A) is represented as:

$$curl(\vec{\mathbf{A}}) = \vec{\nabla} \times \vec{\mathbf{A}} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ \mathbf{A}_x & \mathbf{A}_y & \mathbf{A}_z \end{vmatrix} = \left[\frac{\partial \mathbf{A}_z}{\partial y} - \frac{\partial \mathbf{A}_y}{\partial z}\right]\hat{a}_x + \left[\frac{\partial \mathbf{A}_x}{\partial z} - \frac{\partial \mathbf{A}_z}{\partial x}\right]\hat{a}_y + \left[\frac{\partial \mathbf{A}_y}{\partial x} - \frac{\partial \mathbf{A}_x}{\partial y}\right]\hat{a}_z - -(8)$$

And the vector representation of the Cartesian surface elements is given by:

$$\overrightarrow{\mathbf{ds}} = dy\,dz\,\hat{a}_x + dx\,dz\,\hat{a}_y + dx\,dy\,\hat{a}_z - - - -(9)$$

Hence, the dot product of eq.(8) with eq.(9) ,gives the right hand side of eq.(7). Therefore, eq.(7) can be written in a simple form as:

Thus:

$$\oint_L \vec{\mathbf{A}} \cdot \overrightarrow{\mathbf{dl}} = \oint_L \vec{\nabla} \times \vec{\mathbf{A}} \cdot \overrightarrow{\mathbf{ds}} - - - - - - - - - - - - - - - - -(10)$$

The curl operator has the following properties:

(1). The curl of a vector field is another vector field

(2). $\vec{\nabla}\times(\vec{\mathbf{A}}\pm\vec{\mathbf{B}})=\vec{\nabla}\times\vec{\mathbf{A}}\pm\vec{\nabla}\times\vec{\mathbf{B}})$

(3). $\vec{\nabla}\times(V\,\vec{\mathbf{A}})=V\,(\vec{\nabla}\times\vec{\mathbf{A}})+\vec{\nabla}V\times\vec{\mathbf{A}} \qquad H.W.$

(4). $\vec{\nabla}\times(\vec{\nabla}V)=0$

(5). $\vec{\nabla}\times\vec{\nabla}\times\vec{\mathbf{A}}=\vec{\nabla}(\vec{\nabla}\cdot\vec{\mathbf{A}})-\vec{\nabla}\cdot\vec{\nabla}\vec{\mathbf{A}}=\vec{\nabla}(\vec{\nabla}\cdot\vec{\mathbf{A}})-\nabla^2\vec{\mathbf{A}}$

(6). $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{\mathbf{A}})=0$

(7). $\vec{\nabla}\times(\vec{\mathbf{A}}\times\vec{\mathbf{B}})=\vec{\mathbf{A}}(\vec{\nabla}\cdot\vec{\mathbf{B}})-\vec{\mathbf{B}}(\vec{\nabla}\cdot\vec{\mathbf{A}})+\vec{\mathbf{B}}\cdot\vec{\nabla}\mathbf{A}-(\vec{\mathbf{A}}\cdot\vec{\nabla})\vec{\mathbf{B}}$

Therefore the circulation or the curl of any vectors can be represented mathematically in matrix notation as follows:

$$\vec{\nabla}\times\vec{A}=\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\vec{\nabla}\times\vec{A}=\frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\,\hat{a}_\phi & \hat{a}_z \\ \dfrac{\partial}{\partial \rho} & \dfrac{\partial}{\partial \phi} & \dfrac{\partial}{\partial z} \\ A_\rho & \rho\,A_\phi & A_z \end{vmatrix}$$

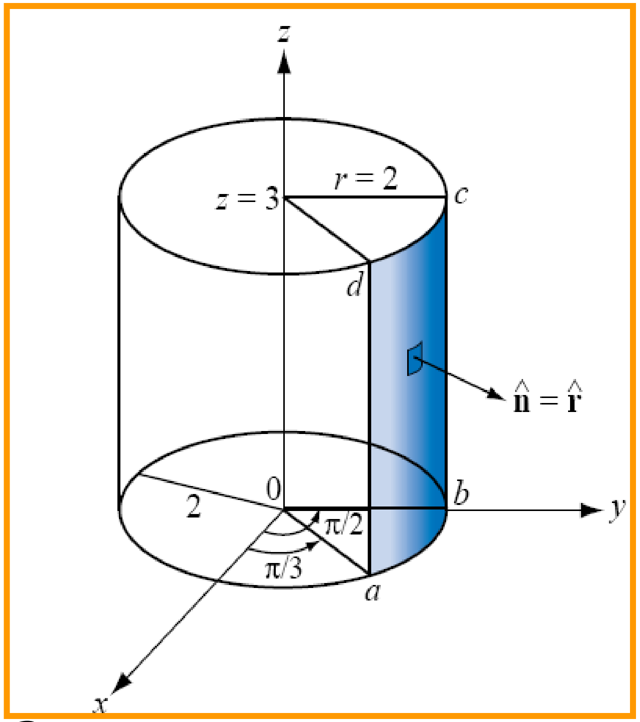
$$\vec{\nabla}\times\vec{A}=\frac{1}{r^2\,\sin\theta}\begin{vmatrix} \hat{a}_r & r\,\hat{a}_\theta & (r\sin\theta)\,\hat{a}_\phi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ A_r & r\,A_\theta & (r\sin\theta)\,A_\phi \end{vmatrix}$$

Example(3): A vector field is given by $\vec{B} = \dfrac{\cos\phi}{\rho}\,\hat{a}_z$ verify Stoke's theorem for a segment of cylindrical surface defined by $r = 2 \quad , \quad 60^\circ \le \phi \le 90^\circ \quad and \quad 0 \le z \le 3$?

Solution:

The mathematical representation of Stokes's theorem is given by: $\oint_L \vec{\mathbf{A}}\cdot\overrightarrow{\mathbf{dl}} = \oint_L \vec{\nabla}\times\vec{\mathbf{A}}\cdot\overrightarrow{\mathbf{ds}}$

The line integral around a closed path defined by these bounded region is as follows:



$$\oint_L \vec{\mathbf{B}}\cdot\overrightarrow{\mathbf{dl}} = \int_a^b \vec{\mathbf{B}}\cdot\overrightarrow{\mathbf{dl}} + \int_b^c \vec{\mathbf{B}}\cdot\overrightarrow{\mathbf{dl}} + \int_c^d \vec{\mathbf{B}}\cdot\overrightarrow{\mathbf{dl}} + \int_d^a \vec{\mathbf{B}}\cdot\overrightarrow{\mathbf{dl}}$$

$$\int_a^b \vec{\mathbf{B}}\cdot\overrightarrow{\mathbf{dl}} = \int \mathbf{B}_z\hat{a}_z \cdot \rho\, d\phi\, \hat{a}_\phi = 0$$

$$\int_b^c \vec{\mathbf{B}}\cdot\overrightarrow{\mathbf{dl}} = \int \mathbf{B}_z\hat{a}_z \cdot dz\,\hat{a}_z = \int_0^3 \frac{\cos\phi}{\rho}dz = \frac{\cos 90}{2}\times z\Big|_0^3 = 0$$

$$\int_c^d \vec{\mathbf{B}}\cdot\overrightarrow{\mathbf{dl}} = \int \mathbf{B}_z\hat{a}_z \cdot \rho\, d\phi(-\hat{a}_\phi) = 0$$

$$\int_d^a \vec{\mathbf{B}}\cdot\overrightarrow{\mathbf{dl}} = \int \mathbf{B}_z\hat{a}_z \cdot dz\,(-\hat{a}_z) = \int_0^3 \frac{\cos\phi}{\rho}dz = -\frac{\cos 60}{2}\times z\Big|_0^3 = -\frac{3}{4}$$

The left hand side of the Stokes's theorem is : $\oint_s \vec{\nabla}\times\vec{\mathbf{A}}\cdot\overrightarrow{\mathbf{ds}}$

$$\vec{\nabla}\times\vec{\mathbf{B}} = \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\,\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ 0 & 0 & \frac{\cos\phi}{\rho} \end{vmatrix} = \frac{1}{\rho}\left[\hat{a}_\rho(-\frac{\sin\phi}{\rho}+0)+\rho\,\hat{a}_\phi(0+\frac{\cos\phi}{\rho^2})+\hat{a}_z(0-0)\right]$$

$$\vec{\nabla}\times\vec{\mathbf{B}} = \frac{1}{\rho^2}\left[\cos\phi\,\hat{a}_\phi - \sin\phi\,\hat{a}_\rho\right]$$

Hence,

$$\oint_s(\vec{\nabla}\times\vec{\mathbf{B}})\cdot\overrightarrow{\mathbf{ds}} = \oint_s \frac{1}{\rho^2}\left[\cos\phi\,\hat{a}_\phi - \sin\phi\,\hat{a}_\rho\right]\cdot\rho\,dz\,d\phi\,\hat{a}_\rho = \int_{\pi/3}^{\pi/2}\int_0^3 -\frac{\sin\phi}{\rho^2}\rho\,dz\,d\phi$$

$$\oint_s(\vec{\nabla}\times\vec{\mathbf{B}})\cdot\overrightarrow{\mathbf{ds}} = -\frac{1}{\rho}\times(\cos\phi)\Big|_{\pi/3}^{\pi/2}\times(z)\Big|_0^3 = \frac{1}{2}\times(\cos\pi/2-\cos\pi/3)\times(3-0)$$

$$\oint_s(\vec{\nabla}\times\vec{\mathbf{B}})\cdot\overrightarrow{\mathbf{ds}} = \frac{3}{2}(0-\frac{1}{2}) \quad\Rightarrow \oint_s(\vec{\nabla}\times\vec{\mathbf{B}})\cdot\overrightarrow{\mathbf{ds}} = -\frac{3}{4}$$

It is clearly seen that the left and right hand side has the same value, which indicates the validity of the Stokes's theorem.

Home Work

Q_1/ Evaluate both sides of the Stoke's theorem on the spherical cap of $r = 2\ m\ ,\quad 0 < \theta < 45^\circ\ , 0 < \phi < 360^\circ$ and it's perimeter, for the field : $\vec{F} = 5r\sin\theta\cos^2\phi\ \hat{a}_\phi$

Q_2/ Find the divergence and curl of the following vector fields:

(a). $\vec{A} = e^{xy}\ \hat{a}_x + \sin xy\ \hat{a}_y + \cos^2 xz\ \hat{a}_z$

(b). $\vec{B} = \rho z^2 \cos\phi\,\hat{a}_\rho + z\sin^2\phi\ \hat{a}_z$

(c). $\vec{C} = r\cos\theta\ \hat{a}_r - \dfrac{1}{r}\sin\theta\ \hat{a}_\theta\ + 2r^2\ \sin\theta\ \hat{a}_\phi$

Q_3/ Find the circulation of each of the following vector fields and evaluate it at its indicated point:

(a). $\vec{A} = 12\sin\theta\ \hat{a}_\theta \qquad at \quad (3, 30^\circ, 0^\circ)$

(b). $\vec{A} = 10\,e^{-2\rho}\cos\phi\ \hat{a}_\rho + 10\sin\phi\ \hat{a}_z \qquad at \quad (2, 0^\circ, 3)$

Q_4/ Verify Stoke's theorem for the vector field : $\vec{A} = (\rho\cos\phi\,\hat{a}_\rho + \sin\phi\,\hat{a}_\phi)$

, in the region defined by: $\rho = 2m\ ,\ z = 0\ ,\ \ and\ \ 0 \le \phi \le \pi$

2-4: Laplacian of a Vector:

For practical reasons, it is expedient to introduce a single operator which is the composite of gradient and divergence operator. This operator is known as the Laplacian.

The Laplacian of a scalar field (V) , written as $\nabla^2 V$ is the divergence of the gradient of (V) $\vec{\nabla} \cdot \vec{\nabla} V$

Thus in Cartesian, Cylindrical and Spherical coordinate systems the Laplacian operator is expressed as:

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cartesian coordinate

$$\nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical coordinate

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 V}{\partial \phi^2}$$

Spherical coordinate

(1). A scalar field (V) is said to be harmonic in a given region if its Laplacian vanishes in that region. In other words, if: $\nabla^2 V = 0$

Then, the solution for (V) is harmonic (it is of the form of the sine and cosine).

(2). The Laplacian operator of a vectors is defined as a grad div (A) $\vec{\nabla}(\vec{\nabla} \cdot \vec{\mathbf{A}})$ and is expressed as:

$$\nabla^2 \vec{\mathbf{A}} = \vec{\nabla}(\vec{\nabla} \cdot \vec{\mathbf{A}}) - \vec{\nabla} \times \vec{\nabla} \times \vec{\mathbf{A}}$$

$$\nabla^2 \vec{\mathbf{A}} = \nabla^2 \mathbf{A}_x \, \hat{a}_x + \nabla^2 \mathbf{A}_y \, \hat{a}_y + \nabla^2 \mathbf{A}_z \, \hat{a}_z$$

Home Work: Prove That: $\vec{\nabla} \times \vec{\nabla} \times \vec{\mathbf{A}} = \vec{\nabla}(\vec{\nabla} \cdot \vec{\mathbf{A}}) - \nabla^2 \vec{\mathbf{A}}$

2-5: Classification of Vector Field:

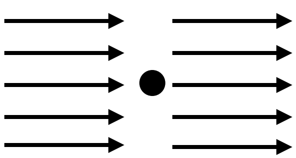
A vector field is uniquely characterized by its divergence and curl. All vector fields can be classified in terms of their vanishing or non-vanishing divergence or curl as follows:

(a). $\vec{\nabla}\cdot\vec{\mathbf{A}}=0 \quad , \quad \vec{\nabla}\times\vec{\mathbf{A}}=0$

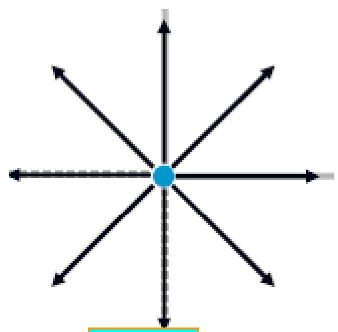
(b). $\vec{\nabla}\cdot\vec{\mathbf{A}}\neq 0 \quad , \quad \vec{\nabla}\times\vec{\mathbf{A}}=0$

(c). $\vec{\nabla}\cdot\vec{\mathbf{A}}=0 \quad , \quad \vec{\nabla}\times\vec{\mathbf{A}}\neq 0$

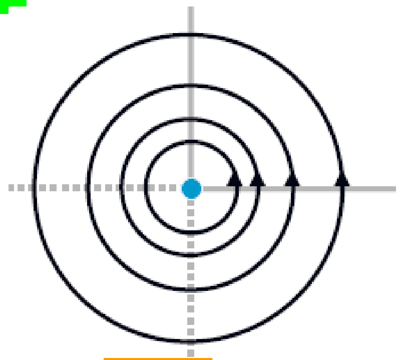
(d). $\vec{\nabla}\cdot\vec{\mathbf{A}}\neq 0 \quad , \quad \vec{\nabla}\times\vec{\mathbf{A}}\neq 0$



(a).



(b).



(c).

(1). A vector field $\vec{\mathbf{A}}$ **is said to be Solenoid (or divergence less) if (** $\vec{\nabla}\cdot\vec{\mathbf{A}}=0$ **) [such field has neither source nor sink of flux] . Example of such fields are (magnetic field, conduction current density under steady state condition).**

The Solenoid field $\vec{\mathbf{A}}$ can always be expressed in terms of another vector such as $\vec{\mathbf{F}}$

$$\vec{\nabla}\cdot\vec{\mathbf{A}}=0 \qquad \textit{means that} \qquad \oint_S \vec{\mathbf{A}}\cdot\overrightarrow{\mathbf{ds}}=0 \qquad \textit{and} \qquad \vec{\mathbf{F}}=\vec{\nabla}\times\vec{\mathbf{A}} \qquad \vec{\nabla}\cdot\vec{\nabla}\times\vec{\mathbf{A}}=0$$

(2). A vector field $\vec{\mathbf{A}}$ is said to be irrotational (or conservative) if, $\vec{\nabla} \times \vec{\mathbf{A}} = 0$

, example of such fields are [Electrostatic field and gravitational field].

$$\vec{\nabla} \times \vec{\mathbf{A}} = \oint_L \vec{\mathbf{A}} \cdot \overrightarrow{\mathbf{dl}} = \oint \vec{\nabla} \times \vec{\mathbf{A}} \cdot \overrightarrow{\mathbf{ds}} = 0$$

Thus, an irritation field $\vec{\mathbf{A}}$ can always be expressed in terms of a scalar field (V), that is if:

$$\vec{\nabla} \times \vec{\mathbf{A}} = 0 \quad \textit{then} \quad \vec{\mathbf{A}} = -\vec{\nabla} V \quad \textit{then} \quad \oint_L \vec{\mathbf{A}} \cdot \overrightarrow{\mathbf{dl}} = 0 \quad \textit{and hence} \quad \vec{\nabla} \times \vec{\nabla} V = 0$$

(3). When $\nabla^2 V = 0$, the field is said to be harmonic

(4). When the field is independent on position the field is said to be uniform field

(5). When the field is independent on time the field is said to be constant field.

(6). The field is said to be Rotational (or non-conservative) if $\vec{\nabla} \times \vec{\mathbf{A}} \neq 0$

(7). The field is said to be divergence if $\vec{\nabla} \cdot \vec{\mathbf{A}} = 0$

$$\frac{d^2y}{dx^2} = \frac{1}{c^2}\frac{d^2y}{dt^2}$$

$$\nabla^2\psi + k^2\psi = 0$$

$$if \quad k^2 = 0$$

the equation is called (Laplace's)

$$if \quad k^2 = -quantity$$

the equation is called (Helmholtiz's)

$$if \quad k^2 = +quantity$$

the equation is called (Poison's)

$$if \quad k^2 = cons\tan t \times kinetic\ energy$$

the equation is called (Schrödinger wave equation)

Example(4): display whether the field vector $\vec{E} = r\,e^{-r}\,\hat{a}_r$ is solenoid, conservative or none of them?

Solution:

(1). In order to show that the field is solenoid, it must be satisfy: $\vec{\nabla}\cdot\vec{\mathbf{E}} = 0$

$$\vec{\nabla}\cdot\vec{\mathbf{E}} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,\mathbf{E}_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\mathbf{E}_\theta\sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial\mathbf{E}_\phi}{\partial\phi} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,r\,e^{-r}\right)$$

$$\vec{\nabla}\cdot\vec{\mathbf{E}} = \frac{1}{r^2}(-r^3 + 2r^2)\,e^{-r} \neq 0$$

Therefore the field is not solenoid:

(2). In order to show that the field is conservative, it must be satisfy: $\vec{\nabla}\times\vec{\mathbf{E}} = 0$

$$\vec{\nabla}\times\vec{\mathbf{E}} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\,\hat{a}_\theta & (r\sin\theta)\,\hat{a}_\phi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial\theta} & \dfrac{\partial}{\partial\phi} \\ \mathbf{E}_r & r\,\mathbf{E}_\theta & (r\sin\theta)\,\mathbf{E}_\phi \end{vmatrix} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\,\hat{a}_\theta & (r\sin\theta)\,\hat{a}_\phi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial\theta} & \dfrac{\partial}{\partial\phi} \\ r\,\mathbf{e}^{-r} & 0 & 0 \end{vmatrix}$$

$$\vec{\nabla}\times\vec{\mathbf{E}} = \frac{1}{r^2\sin\theta}\left[\hat{a}_r\,(0-0) + r\,\hat{a}_\theta\,(0-0) + (r\sin\theta)\,\hat{a}_\phi\,(0-0)\right] = 0$$

$$\therefore\ \vec{\nabla}\times\vec{\mathbf{E}} = 0$$

Therefore the field is not conservative:

Home Work

Q_1/ Given field $\vec{A} = 3x^2 y z\,\hat{a}_x + x^3 z\,\hat{a}_y + (x^3 y - 2z)\,\hat{a}_z$ **, it can be said that** $\vec{A}$

a. Harmonic , **b. Divergenceless** , **c. Solenoid** , **d. Rotational** , **e. Conservative.**

Q_2/ If a vector field $\vec{A}$ **is solenoid, which of the following is true :**

a. $\oint_L \vec{A}\cdot dl = 0$ **b.** $\oint_S \vec{A}\cdot ds = 0$ **c.** $\nabla\times\vec{A} = 0$ **d.** $\nabla\times\vec{A} \neq 0$ **e.** $\nabla^2\vec{A} = 0$

Q_3/ If $(\vec{r})$ **is the position vector of point (x,y,z) and** $r = |\vec{r}|$ **then prove that :**

(a). $\nabla(\ln r) = \dfrac{\hat{a}_r}{r}$ **and** **(b).** $\nabla^2(\ln r) = \dfrac{1}{r^2}$

.

Q_4/ Let $\vec{G} = x^3 y^2 z^2\,\hat{a}_x$ **, then calculate:** **a.** $\nabla\times\vec{G}$ **b.** $\nabla\cdot\nabla\times\vec{G}$ **c.** $\nabla\times\nabla\times\vec{G}$ **d.** $\nabla\times\nabla G_x$

Q_5/ Evaluate $\nabla V\ ,\ \nabla\cdot\nabla V\ ,\ and\ \ \nabla\times\nabla V$ **if :a.** $V = 3x^2 y + xz$ **b.** $V = \rho z\cos\phi$ **c.** $V = 4r^2\cos\theta\sin\phi$