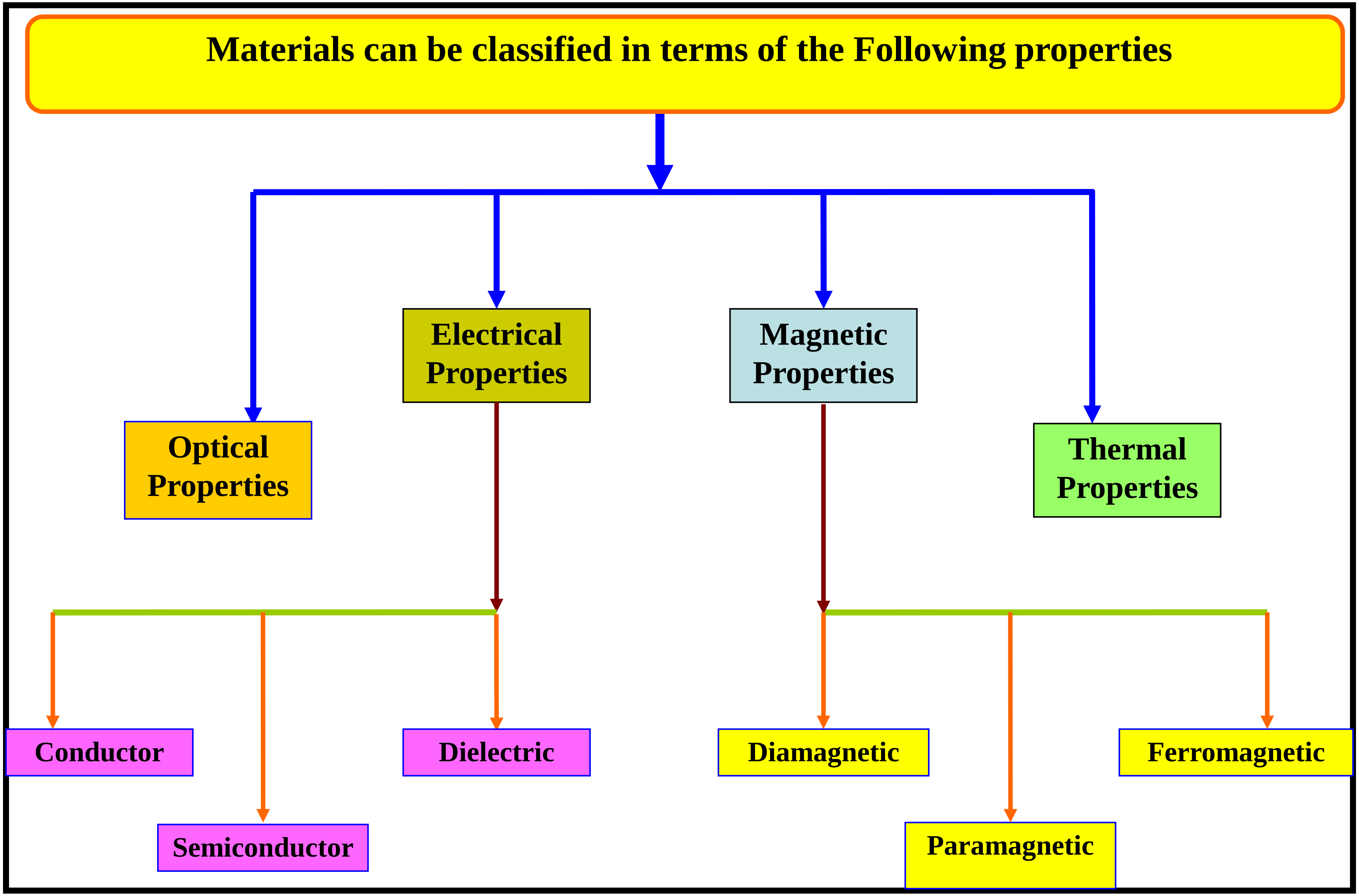
Chapter Four /// Part (I)

Electrostatic Field in Material Space & Boundary Conditions


Materials can be classified in terms of the Following properties
Optical Properties
Electrical Properties
Magnetic Properties
Thermal Properties
Conductor
Semiconductor
Dielectric
Diamagnetic
Paramagnetic
Ferromagnetic

4-1: Introduction:

In chapter three our discussion was restricted to static (stationary) charges in free space (vacuum). Under these conditions we found electrostatic $\vec{\mathbf{E}}, \vec{\mathbf{D}}$ *and* V fields.

In this chapter we will allow the charges to move with a constant velocity and thus introduce the concept of current. The concept of current will tend us to conducting media whose prominent characteristic is that of conducting electric charge. It should be noted that a constant (steady) current will give rise to a constant or steady magnetic field.

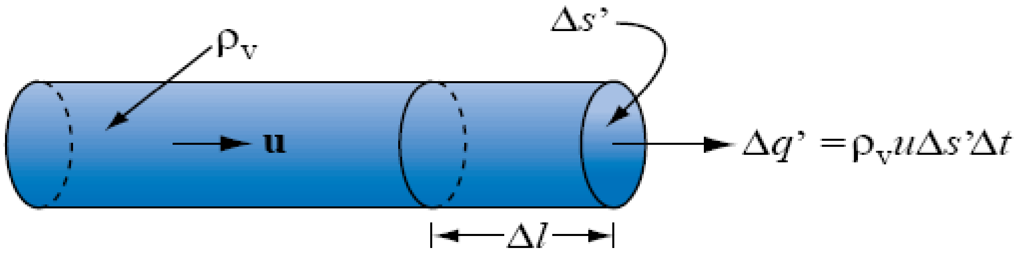
The next logical discussion of this chapter will be that of dielectric media whose prominent characteristic is that of electric polarization formation of electric dipoles with the media.

The introduction of conducting media (conductors) will lead us to the conductivity (σ)and resistance (R)concept, while, the dielectric media (insulators) will lead us to the dielectric constant (permittivity) (ε) and capacitance (C) concept.

The concept of resistance (R) and capacitance (C) are called circuit concepts, since they are very useful in electronic circuit analysis.

4-2: Moving Charges (Current):

Let us assume that we have, in vacuum, a long cylinder of (ρ_v) distribution as shown below. the element of charge (ΔQ), found in the volume $\Delta v = \Delta x \, \Delta s$, is assumed to move with a velocity ($\vec{\mathbf{u}} = \mathbf{u}_x \, \hat{a}_x$) when subjected to the applied field ($\vec{\mathbf{E}} = \mathbf{E}_x \, \hat{a}_x$) then at some time (Δt) the charge of ($\Delta Q = \rho_v \, \Delta v$) will passes through the cross-section surface given by:



$$\frac{\Delta Q}{\Delta t} = \frac{\rho_v \, \Delta v}{\Delta t} = \rho_v \, \frac{\Delta x \; \Delta s}{\Delta t} = \rho_v \, \frac{\Delta x}{\Delta t} \, \Delta s \;\; ------(1)$$

The limit of eq.(1) as ($\Delta t \to 0$) lead us to the definition of the current (ΔI) that flows through the surface (Δs) that is:

$$\Delta I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \lim_{\Delta t \to 0} \rho_v \, \frac{\Delta x}{\Delta t} \, \Delta s = \rho_v \, \mathbf{u}_x \, \Delta s ------(2) \; (A = C/s)$$

Then the current is defined as the movement of charge through a given surface and is equal to the Coulomb per second through that surface.

Let us now divide eq.(2) by (Δs) and take the limit as ($\Delta s \to 0$) to obtain the defining equation for the x-directed current density ($\vec{\mathbf{J}}_x \; (A/m^2)$), thus:

$$\vec{\mathbf{J}}_x = \lim_{\Delta s \to 0} \frac{\Delta I}{\Delta s} = \rho_v \, \vec{\mathbf{u}}_x \quad ----------(3)$$

Eq.(3) can be written in general vector form as: $\vec{\mathbf{J}} = \rho_v \, \vec{\mathbf{u}} \quad ---------(4)$

Here, ($\vec{\mathbf{J}}$) is called convection current density and denoted by ($\vec{\mathbf{J}}_v$).

Therefore, the total current flowing through the large perpendicular cross section of area (S) can be written as:

$$I = \oint_s \vec{\mathbf{J}} \cdot \overrightarrow{\mathbf{ds}} \quad -------(5)$$

Notes:

(1). If (ρ_v) is positive and $\vec{\mathbf{E}} = \mathbf{E}_x\, \hat{a}_x \quad then \quad \vec{\mathbf{u}} = \mathbf{u}_x\, \hat{a}_x \quad and \;\; then \quad \vec{\mathbf{J}} = \mathbf{J}_x\, \hat{a}_x$

(2).Also when (ρ_v) is negative and $\vec{\mathbf{E}} = \mathbf{E}_x\, \hat{a}_x \quad then \quad \vec{\mathbf{u}} = -\mathbf{u}_x\, \hat{a}_x \quad and \;\; then \quad \vec{\mathbf{J}} = \mathbf{J}_x\, \hat{a}_x$

, since

$$\vec{\mathbf{J}} = \rho_v\, \vec{\mathbf{u}} = (-\rho_v) \times (-\mathbf{u}_x\, \hat{a}_x) = \rho_v\, \mathbf{u}_x\, \hat{a}_x = \mathbf{J}_x\, \hat{a}_x$$

(3). On the other hand, when both positive and negative charges are present as in semiconductor, then:

$$\vec{\mathbf{J}} = \rho_{v+}\, \vec{\mathbf{u}}_+ + \rho_{v-}\, \vec{\mathbf{u}}_- \;------------(6)$$

$$\rho_{v+} = \rho_h = n\,h \qquad and \qquad \rho_{v-} = \rho_e = n\,e \;---------(7)$$

4-3: Conductor and Conductivity:

A conducting material (medium) has electric charge conduction as its prominent characteristic. A metal conductor has enormous electric charge conduction through the presence of a large number of outer-orbit electrons that are free to move about in the lattice structures of the material.

These free electrons with ($\vec{\mathbf{E}}_{ext.}=0$) moves randomly with varying velocities to produce zero net current through any surface in the conductor.

However, when applying an external E-field, the electrons will still have motion in random direction but will drift slowly in the ($\vec{\mathbf{E}}-direction$) with a velocity ($\vec{\mathbf{u}}_d$). The motion of electrons in the ($\vec{\mathbf{E}}-direction$ with a velocity ($\vec{\mathbf{u}}_d$) give rise to a conduction current in the conductor.

$$\vec{\mathbf{u}}_d \;\alpha -\vec{\mathbf{E}} \quad \Rightarrow \vec{\mathbf{u}}_d = -\mu_e \vec{\mathbf{E}} --------(1)$$

where (μ_e) is electron mobility in unit ($\frac{m^2}{V\,s}$)

And we have: $\vec{\mathbf{J}} = \rho_v \vec{\mathbf{u}} ---------(2)$, substituting eq.(1) into eq.(2) we get:

$$\vec{\mathbf{J}} = \rho_v(-\mu_e \vec{\mathbf{E}}) = \sigma \vec{\mathbf{E}} ---------(3)$$

since, $\sigma = \mu_e \rho_v$ *is called conductivity* $(S/m)\, or\, (\Omega^{-1} m^{-1})$

When an ($\vec{\mathbf{E}}-field$) is applied, the force on an electron with charge (-e) is:

$$\vec{\mathbf{F}} = -e\vec{\mathbf{E}} -----------(4)$$

If the electron with mass (m) is moving in an electric field with an average drift velocity ($\vec{\mathbf{u}}_d$), then according to Newton's second law:

$$\frac{m\vec{\mathbf{u}}_d}{\tau} = -e\vec{\mathbf{E}} \qquad \Rightarrow\Rightarrow \vec{\mathbf{u}}_d = \frac{e\tau}{m}\vec{\mathbf{E}} ------------(5)$$

Where, (τ) is the average time interval between collisions. This indicates that the drift velocity of the electron is directly proportional to the applied field. If there are (n-electrons) per unit volume, the electronic charge density is given by:

$$\rho_v = -ne ------------(6)$$

thus the conduction current density is given by:

$$\vec{\mathbf{J}} = \rho_v \vec{\mathbf{u}}_d = (-ne)\,(-\frac{e\tau}{m})\vec{\mathbf{E}} = \frac{ne^2\tau}{m}\vec{\mathbf{E}} --------(7)$$

$$\vec{\mathbf{J}} = \sigma\vec{\mathbf{E}} --------(8)$$

$$\text{since} \quad \sigma = \frac{ne^2\tau}{m}$$

$$\sigma = -\mu_e\rho_v \quad hence \quad \sigma = ne\mu_e$$

$$and \quad we \quad have \quad \mu_e = \frac{e\tau}{m}$$

Hence, the conductivity of the conductor $\sigma = \dfrac{ne^2\tau}{m}$

$\vec{\mathbf{J}} = \sigma\vec{\mathbf{E}}$ **is called a point form of Ohm's Law**

There are three types of current densities named (Convection, conduction and displacement currents).

(1). Convection Current Density ($\vec{\mathbf{J}}_v = \rho_v \vec{\mathbf{u}}_d$): is defined as a current which produced due to a movement of charged particle through a vacuum, air, or non-conductive media such as (beam of electron in a cathode ray tube or TV-screen).

(2). Conduction Current Density ($\vec{\mathbf{J}}_c = \sigma \vec{\mathbf{E}}$): is defined as a current which produced due to a movement of electrons through conductive media in response to an applied electric field such as (the flow of current in Copper wires). It is given by the point form of Ohm's law as:

(3). Displacement Current Density ($\vec{\mathbf{J}}_d = \frac{\partial \vec{\mathbf{D}}}{\partial t}$): is defined as the time varying electric field phenomenon that allows current to flow between the plates of a capacitance.

Example(1):

(a). Find the mobility of the conduction electrons of Aluminum with conductivity $\sigma = 3.82\times10^{7}\ (S/m)$ **and conduction electron density** $N_e = 1.7\times10^{29}\ m^{-3}$

(b). Find the concentration of holes N_h **in P-type Germanium, where** $\sigma = 10^{4}\ (S/m)$ **and the hole mobility is** $\mu_h = 0.18\ (m^2/sV)$ **?**

Solution:

(a). the mobility is calculated through: $\sigma = -\mu_e\,\rho_v \quad and\ \rho_v = N_e\ e \quad hence \quad \sigma = N_e\,e\,\mu_e$

Therefore, $\mu_e = \dfrac{\sigma}{N_e\,e} = \dfrac{3.82\times10^{7}}{1.7\times10^{29}\times1.6\times10^{-19}} \Rightarrow\Rightarrow \boxed{\mu_e = 0.0014\ \ (m^2.V^{-1}.s^{-1})}$

(b). $\sigma = \mu_h\,\rho_{v+} \quad and \quad \rho_{v+} = N_h\ e \quad hence \quad \sigma = N_h\,e\,\mu_h$ **Thus,** $N_h = \dfrac{\sigma}{e\,\mu_h}$

Hence, $N_h = \dfrac{\sigma}{e\,\mu_h} = \dfrac{10^{4}}{1.6\times10^{-19}\times0.18} \Rightarrow\Rightarrow \boxed{N_h = 3.47\times10^{25}\ \ m^{3}}$

4-4: Resistance:

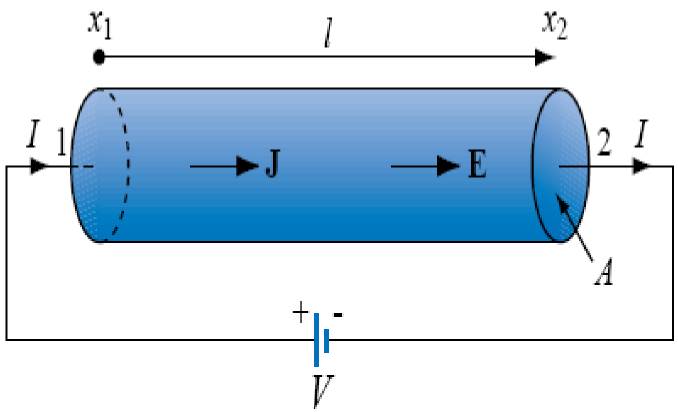
The relationship between the potential difference and the current is the resistance given by the version of Ohm's law ($\mathbf{R} = \frac{\mathbf{V}}{\mathbf{I}}$). Now using the point form of Ohm's law ($\vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$) to derive an expression for the resistance (R) of a conductor of length (l) and uniform cross section (A) as shown below:

When a voltage (V) is applied across the conductor terminals establishes an electric field given by ($\vec{\mathbf{E}} = \mathbf{E}_x \, \hat{a}_x$) and the relation between the potential difference (V) and electric field intensity (E) is given by:

$$\int_{V_1}^{V_2} dV = -\int_{x_1}^{x_2} \vec{\mathbf{E}} \cdot \overrightarrow{\mathbf{dl}} \Rightarrow\Rightarrow V_2 - V_1 = \mathbf{E}(x_2 - x_1) \Rightarrow\Rightarrow V = \mathbf{E} \; l \text{ ---------- } (1)$$

The current flowing through the cross section (A) at (x2) is given as:

$$I = \int_S \vec{\mathbf{J}} \cdot \overrightarrow{\mathbf{ds}} = \int_S \sigma \, \vec{\mathbf{E}} \cdot \overrightarrow{\mathbf{ds}} \Rightarrow\Rightarrow \quad I = \sigma \; \mathbf{E} \; A \text{ ------------ } (2)$$



Dividing eq.(1) by eq.(2) we obtain a resistance of the conductor and is given as:

$$\mathbf{R} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{\mathbf{E}\, l}{\sigma\ \mathbf{E}\ A} = \frac{l}{\sigma\ A} \quad ------------(3)$$

Now we generalize our result for (R) to any resistance of arbitrary shape as:

$$\mathbf{R} = \frac{-\int \vec{\mathbf{E}} \cdot \overrightarrow{\mathbf{dl}}}{\int \vec{\mathbf{J}} \cdot \overrightarrow{\mathbf{d}s}} = \frac{-\int \vec{\mathbf{E}} \cdot \overrightarrow{\mathbf{dl}}}{\sigma \int \vec{\mathbf{E}} \cdot \overrightarrow{\mathbf{d}s}} \quad ------------(4)$$

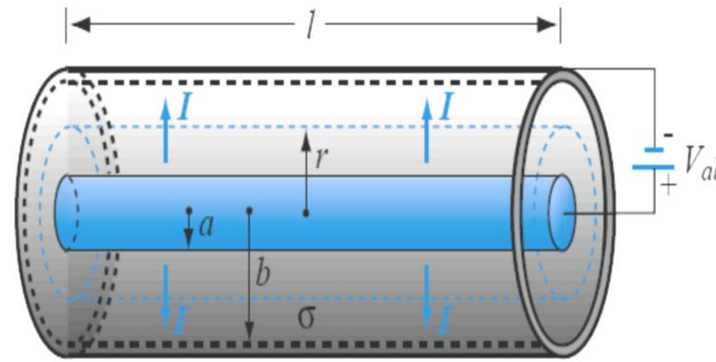
The reciprocal of (R) is called the conductance (G) and the unit of (G) is (Ω-1) or siemens (S). for the linear uniform cross sectional area of a given conductors the conductance is given by:

$$\mathbf{G} = \frac{1}{\mathbf{R}} = \frac{\sigma\ \mathbf{A}}{l} \quad ---------------(5)$$

Example(2): Find the resistance and conductance between the inner conductive shell of radius (a) and outer conductive shell of radius (b) with a length (L) of a coaxial cable filled with material of conductivity (σ).

Solution:

$$\mathbf{R} = \frac{\mathbf{V}_{ab}}{\mathbf{I}} = \frac{-\int \vec{\mathbf{E}} \cdot \overrightarrow{\mathbf{dl}}}{\int \vec{\mathbf{J}} \cdot \overrightarrow{\mathbf{d}s}} = \frac{-\int \vec{\mathbf{E}} \cdot \overrightarrow{\mathbf{dl}}}{\sigma \int \vec{\mathbf{E}} \cdot \overrightarrow{\mathbf{d}s}} ---------(1)$$



For a coaxial cable of the form of cylinder the electric field intensity for charges on the inner shell is given by (see Gauss' law application):

$$\vec{\mathbf{E}} = \frac{Q}{2\pi\varepsilon_{\circ}\rho L}\hat{a}_{\rho}$$

Hence the potential difference is calculated as:

$$V_{ab} = -\int \vec{\mathbf{E}} \cdot \overrightarrow{\mathbf{dl}} = -\int_{b}^{a} \frac{Q}{2\pi\varepsilon_{\circ}\rho}\hat{a}_{\rho} \cdot \overrightarrow{\mathbf{d}\rho}\,\hat{a}_{\rho} = \frac{Q}{2\pi\varepsilon_{\circ} L}\left(\ln(\frac{b}{a})\right) --------(2)$$

And the current can be calculated from the equation: $\mathbf{I} = \int_S \vec{\mathbf{J}} \cdot \overrightarrow{\mathbf{ds}}$

$$\mathbf{I} = \sigma \int_0^{2\pi} \int_0^{L} \vec{\mathbf{E}} \cdot \overrightarrow{\mathbf{ds}} = \sigma \int_0^{2\pi} \int_0^{L} \frac{Q}{2\pi \varepsilon_\circ \rho L} \hat{a}_\rho \cdot \rho \overrightarrow{\mathbf{d}\phi} \overrightarrow{\mathbf{d}z} \hat{a}_\rho = \frac{\sigma Q}{2\pi \varepsilon_\circ L} \int_0^{2\pi} d\phi \int_0^{L} dz$$

hence, $\mathbf{I} = \dfrac{\sigma Q}{\varepsilon_\circ} - - - - - - -(3)$

Substituting eqs.(2) and (3) into eq.(1) we get: $\mathbf{R} = \dfrac{\dfrac{Q}{2\pi \varepsilon_\circ L}\left(\ln(\dfrac{b}{a})\right)}{\dfrac{\sigma Q}{\varepsilon_\circ}} = \dfrac{1}{2\pi \sigma L}\left(\ln(\dfrac{b}{a})\right)$

and the conductance is given by: $\mathbf{G} = \dfrac{1}{\mathbf{R}} = \dfrac{2\pi \sigma L}{\left(\ln(\dfrac{b}{a})\right)}$

And the conductance per unit length is given by: $\mathbf{G}' = \dfrac{\mathbf{G}}{L} = \dfrac{2\pi \sigma}{\left(\ln(\dfrac{b}{a})\right)}$

4-5: Joule's Law:

The electric field does work in moving charges through a material. Some of the energy of the moving charges is given up in collisions with atoms of the material. The amount of energy given up per unit time is called the dissipated power (P).

The differential force exerted by the electric field to move a differential Charge $dQ = \rho_v \, dv$ is:

$$\overrightarrow{\mathbf{dF}} = \vec{\mathbf{E}}\, dQ = \rho_v \, \vec{\mathbf{E}}\, dv \text{-------}(1)$$

The incremental work done is simply given by:

$$dW = \overrightarrow{\mathbf{dF}} \cdot \overrightarrow{\mathbf{dl}} = \rho_v \, dv \, \vec{\mathbf{E}} \cdot \overrightarrow{\mathbf{dl}} \text{-------}(2)$$

The increment of power dissipated is this work divided by the increment of time or is given by:

$$dP = \frac{dW}{dt} = \rho_v \, dv \, \vec{\mathbf{E}} \cdot \frac{\overrightarrow{\mathbf{dl}}}{dt} = \rho_v \, dv \, \vec{\mathbf{E}} \cdot \vec{\mathbf{u}} \text{-------}(3)$$

Rearranging eq.(3) gives: $$dP = \frac{dW}{dt} = \rho_v \, dv \, \vec{\mathbf{E}} \cdot \frac{\overrightarrow{\mathbf{dl}}}{dt} = \rho_v \, \vec{\mathbf{u}} \cdot \vec{\mathbf{E}} \;\; dv \text{-------}(4)$$

But $$\vec{\mathbf{J}} = \rho_v\, \vec{\mathbf{u}} ---------------(5)$$

Now substituting eq.(5) into eq.(4) we obtain:

$$dP = \vec{\mathbf{J}} \cdot \vec{\mathbf{E}}\ \ dv = \sigma\, \vec{\mathbf{E}} \cdot \vec{\mathbf{E}}\ \ dv$$

Finally, we can integrate this equation over the volume to find the total dissipated power as:

$$\mathbf{P} = \int_v \sigma\, \vec{\mathbf{E}} \cdot \vec{\mathbf{E}}\ \ dv = \sigma \int_v \mathbf{E}^2\ \ dv -----------(5)$$

Equation (5) is known as Joule's law, which is describe the power dissipated inside material in the form of heat.

4-6: Continuity Equation & Relaxation Time:

Due to the principle of charge conservation, the time rate of decrease of charge within a given volume must equal to the net outward current flow through the closed surface of the volume. Thus current (I_{out}) coming out of the closed surface is given by:

$$\mathbf{I}_{out} = \oint_S \vec{\mathbf{J}} \cdot \overrightarrow{\mathbf{ds}} = -\frac{dQ_{in}}{dt} \text{-------------} (1)$$

Where, (Qin) is the total charge enclosed by the closed surface. Applying divergence theorem to eq.(1) we get:

$$\oint_S \vec{\mathbf{J}} \cdot \overrightarrow{\mathbf{ds}} = \int_v (\vec{\nabla} \cdot \vec{\mathbf{J}})\, dv \text{---------} (2) \qquad and \qquad dQ = \int_v \rho_v \, dv \text{-------} (3)$$

Substituting eqs.(2) and (3) into eq.(1) we get:

$$\int_v (\vec{\nabla} \cdot \vec{\mathbf{J}})\, dv = -\frac{\partial}{\partial t} \int_v \rho_v \, dv$$

and hence:

$$\vec{\nabla} \cdot \vec{\mathbf{J}} = -\frac{\partial \rho_v}{\partial t} \text{----} (4)$$

or it can be written as:

$$\vec{\nabla} \cdot \vec{\mathbf{J}} + \frac{\partial \rho_v}{\partial t} = 0 \text{---------} (5)$$

eqs.(4) and (5) are called current continuity equation or equation of the conservation of charges.

For steady currents $\frac{\partial \rho_v}{\partial t} = 0$ and hence $\vec{\nabla} \cdot \vec{\mathbf{J}} = 0$

Eq.(4) shows that the total charge leaving a volume is the same as the total charge emanating from it.

The physical meaning of eq.(4) is that [The divergence of the current flux emanating from a point is equal to the time rate decrease of volume charge density at the same point].

In conjunction with Ohm's law: $\vec{\mathbf{J}} = \sigma \vec{\mathbf{E}} --------(6)$

and Gauss's law: $\vec{\nabla} \cdot \vec{\mathbf{E}} = \frac{\rho_v}{\varepsilon_\circ} ---(7)$

Substituting eqs.(6) and (7) into eq.(5) we get:

$$\vec{\nabla} \cdot (\sigma \vec{\mathbf{E}}) = \sigma (\vec{\nabla} \cdot \vec{\mathbf{E}}) = \frac{\sigma}{\varepsilon_\circ} \rho_v = -\frac{\partial \rho_v}{\partial t} \Rightarrow$$

or

$$\frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\varepsilon_\circ} \rho_v = 0$$

$$\int_{\rho_{v\circ}}^{\rho_v} \frac{d\rho_v}{\rho_v} = -\int_0^t \frac{\sigma}{\varepsilon_\circ} dt -------------(8)$$

Integrating equation (8) and rearranging terms we get:

$$\rho_v = \rho_{v\circ}\ \mathbf{e}^{-\frac{\sigma}{\varepsilon_\circ} t} = \rho_{v\circ}\ \mathbf{e}^{-\frac{t}{\tau_r}} \quad ---(9)$$

Home Work: prove that (τr) has a unit of (sec).

Where, ($\tau_r = \frac{\varepsilon_\circ}{\sigma}$) is called relaxation time or rearrangement time. The relaxation timeis defined as: [The time it takes a charge placed in the interior of a material to drop to (e^{-1}) of its initial value].

In eq.(9), ($\rho_{v\circ}$) is the initial charge density (i.e., $\rho_v\ at\ t=0$). This equation shows a decay of charge at some interior point of the material to move toward the surface of the material. On the other hand, the relaxation time can be considered as a parameter which distinguish the conductor materials from dielectric materials as demonstrated below:

(1). For Copper: ($\sigma = 5.8\times10^{7}\ S/m \quad \varepsilon = \varepsilon_\circ = 8.85\times10^{-12}$ **) hence :** $\tau_r = 1.53\times10^{-19}\ \text{sec}$

(2). For Quartz: ($\sigma = 10^{-17}\ S/m \quad \varepsilon_r = 5 \quad \varepsilon = \varepsilon_r\,\varepsilon_\circ = 5\times8.85\times10^{-12}$ **) hence :** $\tau_r = 51.2\ days$

This example indicates that the charge density inside the conductor is equal to zero ($\rho_v = 0$) and hence $\mathbf{E} = 0$

Example(3): The charge $Q = 10^{-4} \times e^{-3t}\ C$ is removed from a sphere through a wire.

Find the current in the wire at $t = 0 \quad and \quad at \quad t = 2.5 \sec$?

Solution:

The current is related to the flow of the charge through a relation:

$\mathbf{I} = -\dfrac{d\mathbf{Q}}{dt}$,Hence, $$\mathbf{I} = -\frac{d(10^{-4} \times \mathbf{e}^{-3t})}{dt} = 3 \times 10^{-4}\, \mathbf{e}^{-3t} \quad ,$$

Therefore, the value of current at ($t = 0$) is:

$$\mathbf{I} = 3 \times 10^{-4}\, \mathbf{e}^{-3t} \;=\; ,3 \times 10^{-4} \ \ Ampere$$

And at ($t = 2.5s$) the value of current is :

$$\mathbf{I} = 3 \times 10^{-4}\, \mathbf{e}^{-3t} \;=\; ,0.0016 \times 10^{-4} \ \ Ampere$$

Home Work

Q$_1$/ A Copper bar with conductivity $\sigma = 5.8\times 10^{7}\ (S/m)\ \ and\ \ mobility\ \ \mu_e = 0.0032\, m^{-3}$ and of rectangular cross-section $0.02\ m\ \ by\ \ 0.08\ m\ \ and\ \ length\ \ 2\ m$ has a voltage drop of $50\ mV$

Find the resistance, current, current density, electric field intensity, and conduction electron drift velocity?

$Ans.[\ 21.6\ \ \mu\Omega,\ 2.32\ kA\ ,\ 1.45\ MA/m^{2}\ \ ,\ 25\ \ mV/m\ ,\ \ 0.08\ \ mm/s\ ]$

Q$_2$/ Find the total current in a circular conductor of radius $2\ m$, if the current density varies with (r), according to :

$$\vec{J} = \frac{10^{3}}{r}\ \hat{a}_r\ \ (A/m^{2})\ \ ?\ \ \ Ans.\ 4\pi\ \ A$$

Q$_3$/ In cylindrical coordinates, $\vec{J} = 10\ e^{-100\,\rho}\ \ \hat{a}_\phi\ \ (A/m^{2})$ for the region: $0.01 \le \rho \le 0.02\ m\ ,\ \ 0 \le z \le 1\ m$

Find the total current crossing the intersection of this region with plane $\phi = constant\ ?\ \ Ans.\ \ 23.3\ \ mA$

Q$_4$/ The relaxation time of Mica with $(\sigma = 10^{-15}\ \ S/m\ \ \ and\ \ \ \varepsilon_r = 6)$ is:

a. $5\times 10^{-10}\ \ \sec$ **b.** $10^{-6}\ \ \sec$ **c.** $5\ \ hours$ **d.** $15\ hours$

Q$_5$/ In a certain region, $\vec{J} = 3r^{2}\ \cos\theta\ \hat{a}_r - r^{2}\sin\theta\ \hat{a}_\theta\ \ (A/m^{2})$ Find the current crossing the surface defined by:

$\theta = 30^{\circ}\ \ ,\ \ 0 \le \phi \le 2\pi\ \ \ and\ \ \ 0 \le r \le 2\ m$?

Q_6/ Determine the total current in a wire of radius $\rho = 1.6\,mm$ if $\vec{J} = \dfrac{500}{\rho}\,\hat{a}_z \;\;(A/m^2)$?

Q7/ Which of the following is not an example of convection current:

a. A moving charged belt

b. Electronic movement in a vacuum tube

c. An electron beam in television tube

d. electric current flowing in a copper wire

Q_8/ Determine the relaxation time for each of the following medium:

a. Hard rubber with $\sigma = 10^{-15}\ S/m \quad and \quad \varepsilon = 3.1\ \varepsilon_\circ$ **b. Mica with** $\sigma = 10^{-15}\ S/m \quad and \quad \varepsilon = 6\ \varepsilon_\circ$

Q_9/ The excess charge in a certain medium decrease to one third ($\frac{1}{3}$)of its initial value in $t = 20\,\mu s$. If the conductivity of the medium is $\sigma = 10^{-4}\ S/m$, what is the dielectric constant of the medium, and the relaxation time ? after $30\,\mu s$ what fraction of the charge will remain?

Q_{10}/ The charge within a spherical volume of radius (2 m) changes linearly from $10^{-9}\,C \;\; to \;\; 10^{-10}\,C \quad in \quad 1\sec$. What is the total convection current that flows outwardly?