Chapter Four /// Part (II)

Electrostatic Field in Material Space & Boundary Conditions

4-7: Dielectric and Polarization:

The fundamental difference between a conductor and dielectric is that a conductor has loosely held (free) electrons that can generate through the crystalline structure of the material, whereas the electrons of the outermost shells of a dielectric are strongly bound to the atom.

When a dielectric material is placed in an electric field, we find that a microscopic distance (displacement) takes place between the average equilibrium positions of the positive and negative bound charges of atoms and molecules. This displacement gives rise to an electric dipole moment ($\vec{\mathbf{P}} = Q\vec{\mathbf{d}}$). These electric dipoles have an effect on the electric field in which the dielectric material is placed.

There are three basic polarization mechanisms that occurs in dielectric materials due to an external electric field; they are:

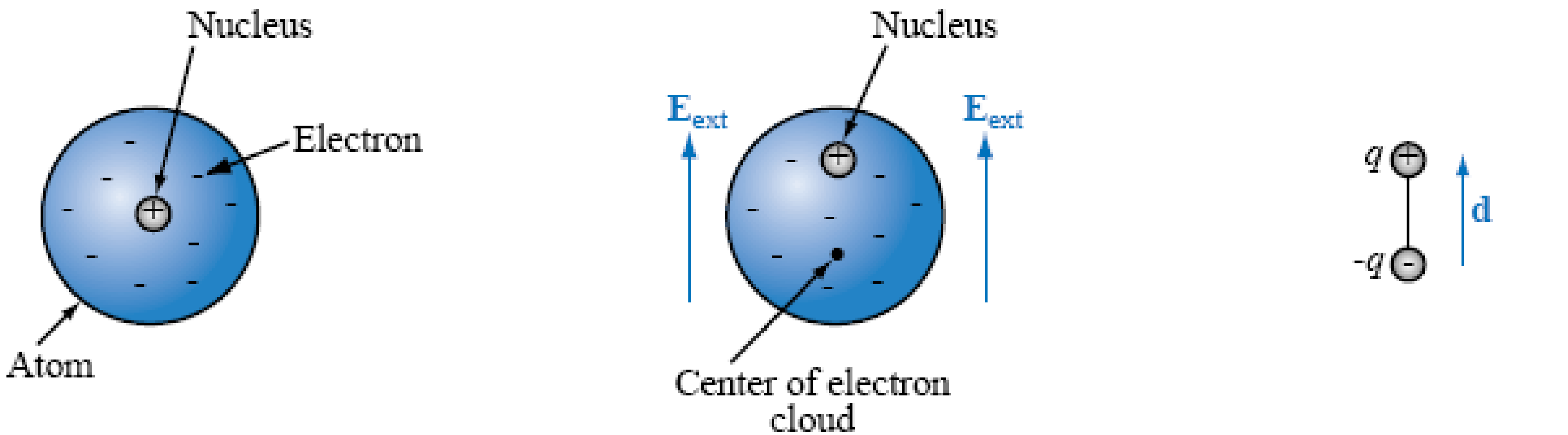
(1). Electronic polarization

(2). Ionic polarization

(3).Orientation polarization.

(1): Electronic Polarization:

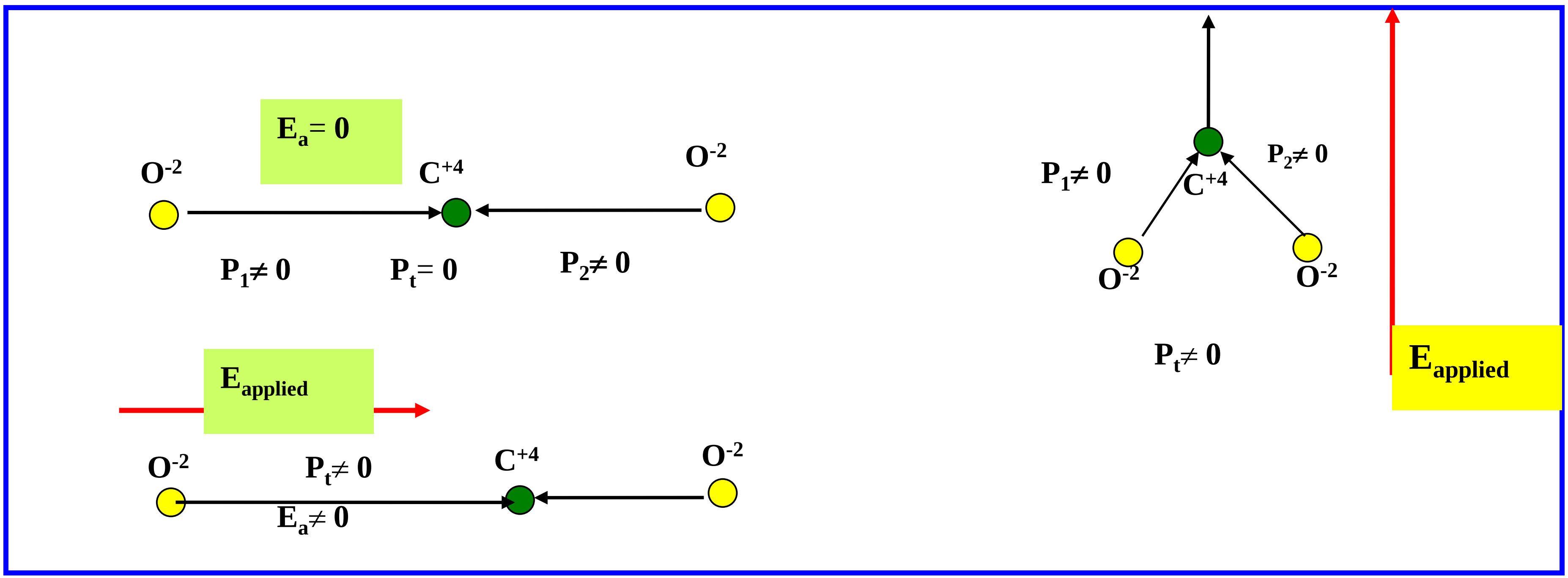
This type of polarization exist in an atom, when placed under the application of an electric field, the center of the cloud of electrons is displaced relative to the center of the nucleus as shown graphically in figure below.



(a) $\mathbf{E}_{ext} = 0$ (b) $\mathbf{E}_{ext} > 0$ (c) Electric dipole

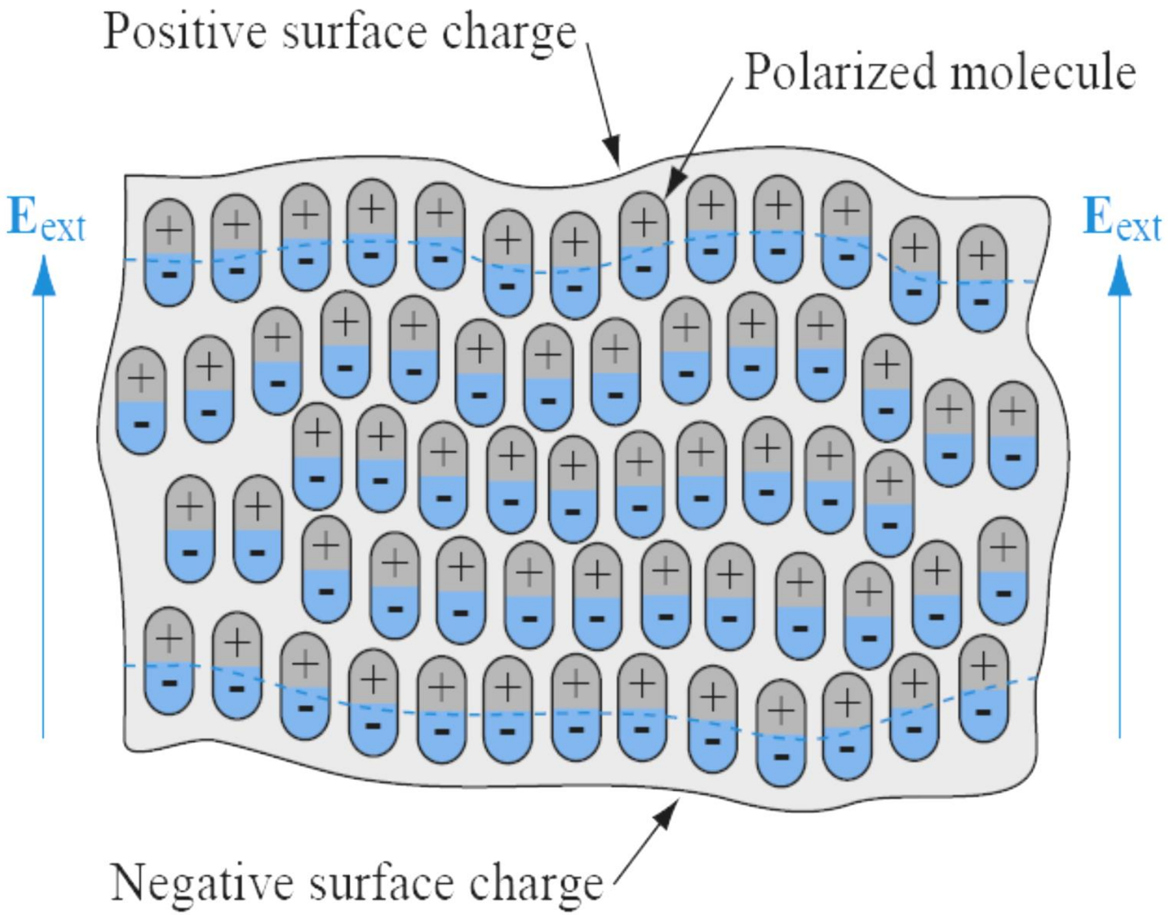
(2): Ionic Polarization:

This type of polarization exists in molecules, having ionic bonds that can be viewed as a build up of positive and negative ions. An applied electric field will displace the positive ions relative to the negative ions. Sodium Chloride (NaCl) and (Co_2) is a good example of this type of polarization and it graphically represented below.



(3): Orientation Polarization:

This type of polarization exist in a material whose molecules posses permanent dipole moments that are randomly oriented in the absence of an applied field, but tend to orient themselves in the direction of an applied electric field. This type of material is also called (Polar Material), water (H2O) being a good example of polar material and it represented graphically in figure below.



Notes:

(1). Dielectric materials that polarized through orientation is called polar material, while materials that polarized through Electronic or Ionic polarization is called non-polar material.

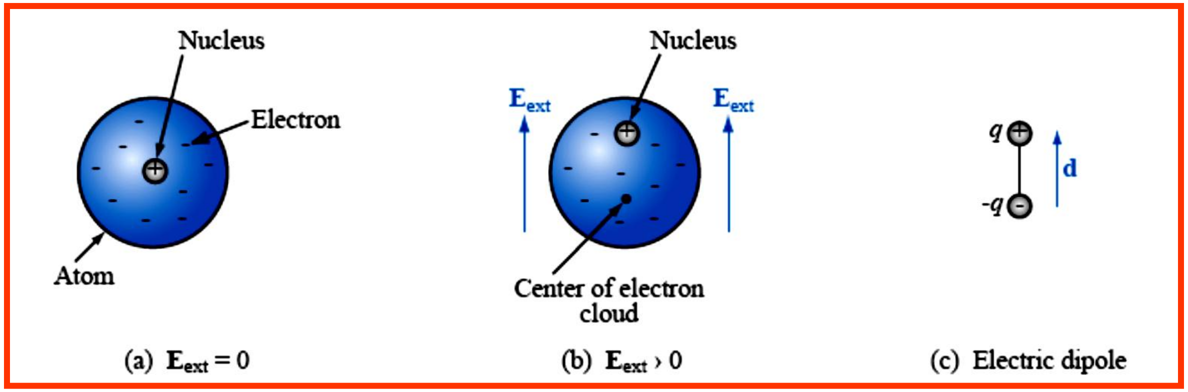
(2). In the case of the orientational (polar) polarization, it should be noted that $\vec{P} \neq 0 \;\; but \;\; \vec{\mathbf{P}}_t = 0$, due to random orientation of $\vec{P} \; when \; \mathbf{E}_{appl.} = 0$

Finally the Torque on the electric dipole moment ($\vec{P}$) that causes the alignment of electric dipole with applied electric field ($\vec{\mathbf{E}}_{appl.}$) is given by:

$$\mathrm{T} = \vec{\mathbf{P}} \times \vec{\mathbf{E}}_{app.} = Q\vec{\mathbf{d}} \times \vec{\mathbf{E}}_{app.}$$

When an electric field ($\vec{\mathbf{E}}$) is applied to a an atom of a given material, the positive charge (nucleus) is displaced from its equilibrium position in the direction of ($\vec{\mathbf{E}}$) by the force ($\vec{\mathbf{F}}_+ = Q\vec{\mathbf{E}}$), while the negative charge (electron) is displaced in the opposite direction by the force ($\vec{\mathbf{F}}_- = Q\vec{\mathbf{E}}$). A dipole results from the displacement of the charges, and the dielectric is said to be polarized as shown below:

$$\vec{P} = Q\,\vec{d}$$



If there are (N) dipoles in a volume (Δv) of the dielectric material, the total dipole moment due to the electric field is given as:

$$\vec{P} = Q_1\,\vec{d}_1 + Q_2\,\vec{d}_2 + Q_3\,\vec{d}_3 + \dots\dots\dots + Q_N\,\vec{d}_N = \sum_{k=1}^{N} Q_k\,\vec{d}_k \text{ -----(1)}$$

As a measure of intensity of the polarization, we define Polarization [$\vec{\mathbf{P}}\,(C/m^2)$] , as the dipole moment per unit volume of the dielectric materials, that is:

$$\vec{\mathbf{P}} = \lim_{\Delta v \to 0}\left(\frac{\vec{P}}{\Delta v}\right) = \lim_{\Delta v \to 0}\left(\frac{\sum_{k=1}^{N} Q_k\,\vec{d}_k}{\Delta v}\right) \text{ -------------(2)}$$

Let us now calculate the field due to polarized dielectric. Consider the dielectric material consisting of dipoles with dipole moment ($\vec{P} = Q\vec{d}$) per unit volume. Then the potential (dV) at an exterior point (O)

due to the dipole moment ($\vec{P}\,dv'$) is:

$$dV = \frac{Q\,\vec{d}\cos\theta}{4\pi\varepsilon_\circ R^2} \quad , \quad \text{since}: \quad \vec{P} = Q\,\vec{d} \quad \text{and} \quad \vec{P}\,dv' = \sum_{k=1}^{N} Q_k \vec{d}_k$$

$$\therefore\ dV = \frac{\vec{P}\cdot\hat{a}_R\,dv'}{4\pi\varepsilon_\circ R^2} \quad ----------(3)$$

$$\vec{\mathbf{R}} = (x-x')\,\hat{a}_x + (y-y')\,\hat{a}_y + (z-z')\,\hat{a}_z \quad \text{and} \quad R = \left|\vec{\mathbf{R}}\right| = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} \quad \hat{a}_R = \frac{\vec{\mathbf{R}}}{\left|\vec{\mathbf{R}}\right|}$$

And we have the identity: $\nabla'(\frac{1}{R}) = \frac{\hat{a}_R}{R^2} --------(4)$ **prove this (H.W.)**

$$\nabla'(\frac{1}{R}) = \left[\frac{d}{dx'}\hat{a}_x + \frac{d}{dy'}\hat{a}_y + \frac{d}{dz'}\hat{a}_z\right]\left(\frac{1}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}}\right)$$

$$\nabla'(\frac{1}{R}) = \frac{(x-x')\,\hat{a}_x}{R^3} + \frac{(y-y')\,\hat{a}_y}{R^3} + \frac{(z-z')\,\hat{a}_z}{R^3} = \frac{\vec{\mathbf{R}}}{R^3} = \frac{1}{R^2}\,\frac{\vec{\mathbf{R}}}{R} = \frac{\hat{a}_R}{R^2}$$

Thus,

$$\frac{\vec{P}\cdot\hat{a}_R}{R^2} = \vec{P}\cdot\nabla\left(\frac{1}{R}\right) ---------- (5)$$

Applying the vector identity: $\vec{\nabla}'\cdot(f\,\vec{\mathbf{A}}) = f\,(\vec{\nabla}'\cdot\vec{\mathbf{A}}) + \vec{\mathbf{A}}\cdot(\vec{\nabla}'f)$ **to eq.(5) we obtain:**

$$\frac{\vec{P}\cdot\hat{a}_R}{R^2} = \vec{P}\cdot\vec{\nabla}'\left(\frac{1}{R}\right) = \vec{\nabla}'\cdot\left(\frac{\vec{P}}{R}\right) - \frac{1}{R}(\vec{\nabla}'\cdot\vec{P}) -------- (6)$$

Substituting eq.(6) into eq.(3) we get:

$$V = \frac{1}{4\pi\varepsilon_\circ}\int_v \left(\vec{\nabla}'\cdot\left(\frac{\vec{P}}{R}\right) - \frac{1}{R}(\vec{\nabla}'\cdot\vec{P})\right) dv' ----------- (7)$$

Applying divergence theorem to the first term of eq.(7) leads finally to:

$$V = \frac{1}{4\pi\varepsilon_\circ}\int_{s'} \frac{\vec{P}}{R}\cdot\overrightarrow{\mathbf{ds}'} - \frac{1}{4\pi\varepsilon_\circ}\int_{v'} \frac{1}{R}(\vec{\nabla}'\cdot\vec{P})\,dv' ---------- (8)$$

The first term in eq.(8), denote the potential due to surface charge density and the second term represent volume charge density:

$$\left.\begin{aligned} \rho_{\rho_s} &= \vec{P}\cdot\hat{a}_n \\ \rho_{\rho_v} &= -\vec{\nabla}\cdot\vec{P} \end{aligned}\right\} ----------(9)$$

In other words eq.(8) reveals that where polarization occurs, an equivalent volume charge density (ρ_{ρ_v}) is formed throughout the dielectric, while an equivalent surface charge density (ρ_{ρ_s}) formed over the surface of the dielectric as shown In the figure:

(ρ_{ρ_s}): is bound (or polarization) surface charge density

(ρ_{ρ_v}): is bound (or polarization) volume charge density

Bound charges are those that are not free to move within the dielectric material. The total positive charge on surface (S) bounding the dielectric is given as:

$$Q_b = \oint_S \vec{P} \cdot \overrightarrow{\mathbf{ds}} = \int_S \rho_{\rho_s} ds ----------- (10)$$

While the charge that remains inside (S) is:

$$-Q_b = \int_v \rho_{\rho_v} dv = -\int_v (\vec{\nabla} \cdot \vec{P}) \, dv ------ (11)$$

Thus the total charge of the dielectric material remains zero, that is:

$$Total\ ch\arg e = \oint_S \rho_{\rho_s} ds + \int_v (\vec{\nabla} \cdot \vec{P}) \, dv = Q_b - Q_b = 0 -------- (12)$$

We now consider the case in which the dielectric region contains free charge.

If (ρ_v) is the free charge volume density, then the total volume charge density (ρ_t) is given by:

$$\rho_t = \rho_v + \rho_{\rho_v} = \vec{\nabla} \cdot \varepsilon_\circ \vec{\mathbf{E}} ------------ (13)$$

Hence: $\rho_v = \vec{\nabla} \cdot \varepsilon_\circ \vec{\mathbf{E}} - \rho_{\rho_v} = \vec{\nabla} \cdot \varepsilon_\circ \vec{\mathbf{E}} + \vec{\nabla} \cdot \vec{P} -------(14)$

But, $\rho_v = \vec{\nabla} \cdot \vec{\mathbf{D}} ------------------(15)$

Then, $\therefore\ \vec{\nabla} \cdot \vec{\mathbf{D}} = \vec{\nabla} \cdot \varepsilon_\circ \vec{\mathbf{E}} + \vec{\nabla} \cdot \vec{P}$ and $\vec{\mathbf{D}} = \varepsilon_\circ \vec{\mathbf{E}} + \vec{P} ----(16)$

We conclude that the net effect of the dielectric material on the electric field intensity is to increase ($\vec{\mathbf{D}}$) inside it by amount ($\vec{P}$). The polarization ($\vec{P}$) vary directly with the applied electric field given by:

$$\vec{\mathbf{P}} = \chi_e\ \varepsilon_\circ \vec{\mathbf{E}}\ ------(17)$$

Where, (χ_e) is called electric susceptibility and is define as a measure of how susceptible (or sensitive) a given dielectric material to the applied electric field.

Substituting eq.(17) into eq.(16) we get:

$$\vec{\mathbf{D}} = \varepsilon_\circ \vec{\mathbf{E}} + \chi_e \varepsilon_\circ \vec{\mathbf{E}} \Rightarrow\Rightarrow \vec{\mathbf{D}} = \varepsilon_\circ \vec{\mathbf{E}}(1 + \chi_e) ---(18)$$

But, $(\chi_e + 1) = \varepsilon_r \quad and \quad \varepsilon = \varepsilon_r\, \varepsilon_\circ$

where (ε_r **) is called relative permittivity and is given by:** $\varepsilon_r = \dfrac{\varepsilon}{\varepsilon_\circ}$,

where (ε **) is called dielectric constant of the material medium and (** $\varepsilon_\circ$ **) is called free space permittivity and its value is equal to (** $\varepsilon_\circ = \dfrac{10^{-9}}{36\pi}\ (F/m) = 8.85\times10^{-12}\ (F/m)$ **).**

Example(4): In a sample of dielectric , $\vec{P} = 5\times10^{-14}\ \hat{a}_z\ (C/m^2)$ **at a point where** $\vec{E} = 2\ \hat{a}_z\ (V/m)$ **, find** χ_e , ε , ε_r and $\vec{D}$ **?**

Solution:

We have, $\vec{\mathbf{P}} = \varepsilon_\circ\ \chi_e\ \vec{\mathbf{E}} \Rightarrow\Rightarrow \chi_e = \dfrac{\mathbf{P}}{\varepsilon_\circ\ \mathbf{E}} = \quad \chi_e = \dfrac{5\times10^{-14}}{2\times\dfrac{10^{-9}}{36\pi}} \Rightarrow\Rightarrow \chi_e = 90\pi\times10^{-5} = 0.002826$

$\varepsilon_r = \chi_e + 1 = 1.002826 \qquad and \qquad \varepsilon = \varepsilon_\circ\ \varepsilon_r = 8.85\times10^{-12}\times(1.002826) \Rightarrow \varepsilon = 8.87\times10^{-12}\ (F/m)$

And , $\vec{\mathbf{D}} = \varepsilon\ \vec{\mathbf{E}} = 8.87\times10^{-12}\times 2\hat{a}_z \Rightarrow\Rightarrow \qquad \vec{\mathbf{D}} = 17.75\times10^{-12}\ (C/m^2)$

Home work

Q$_1$/ Sea water has $\varepsilon_r = 80$. It's permittivity is :

a. 81 F/m **b.** 79 F/m **c.** 5.162 $*10^{-16}$ F/m **d.** 7.074 $*10^{-10}$ F/m

Q$_2$/ Both $\varepsilon_\circ$ *and* χ_e are dimensionless: **a. True** **b. False**

Q3/ State and derive the point form expression for the conservation of charges?

Q4/ Derive the point form of Ohm's law?

Q5/ What is the relaxation time? Derive the decay-charge equation?