Chapter seven
Magnetostatic field

7-1: Introduction and History:

In the previous chapters, we have studied the electric charges and the fields they produces, which are $\vec{\mathbf{E}}, \vec{\mathbf{D}}$ *and* $\mathbf{V}$. We also shown that moving charges gives rise to the electric current concept.

In this chapter, we shall find that a current will produce a magnetic field that will in turn produce a force on magnetic material, magnets and other current configurations. We shall also show that the magnetic monopole can not be isolated in the same manner as electric charges .

In this chapter we focus our attention to the static magnetic fields, which characterize by

$$\vec{\mathbf{H}} - magnetic\ Field\ \ Intensity\ \ (A/m)$$

$$\vec{\mathbf{B}} - Magnetic\ \ Flux\ Density\ (Tesla)$$

$$\mathbf{B} = \mu\,\mathbf{H} \qquad like \qquad \mathbf{D} = \varepsilon\,\mathbf{E}$$

$$for\ \ free\ \ space\ \ \mu = \mu_\circ = 4\pi \times 10^{-7}\ \ (H/m)$$

1- **ca. BC 600,** In northern Greece, a shepherd named Magnus experiences a pull on the iron nails in his sandals by the black rock he was standing on. The region was later named Magnesia and the rock became known as magnetite [a form of iron with permanent magnetism Fe_3O_4].

2- **ca. 1000,** Magnetic compass used as a navigational device

3- **1820 Hans Christian Oersted (1777-1851)(Danish) after (13) years** of experiments study and efforts, demonstrates the interconnection between electricity and magnetism through his discovery that an electric current in a wire causes a compass needle to orient itself perpendicular to the wire.

4- **1821 Andre-Marie Ampere (French)** notes that parallel currents in wires attract each other and opposite currents repel.

5- **1821 Jean-Baptiste Biot (French) and Felix Savart (French)** develop the Biot-Savart law relating the magnetic field induced by a wire segment to the current flowing through it.

6- **1827 Joseph Henry (American)** introduces the concept of inductance and built one of the earliest electric motors. He also assisted Samuel Morse in the development of the telegraph.

7- **1831 Michael Faraday (English)** discovers that a changing magnetic flux can induce an electromotive force.

7-2: Sources and Applications of Magnetic Field:

The magnetic field can be produces due to one of the following sources:

1- Permanent magnetic material

2- Moving charge with constant velocity (or steady current)

3- Time varying electric fields.

The magnetic fields are used in most of the electric devices such as the following:

1- Motors

2- Transformers

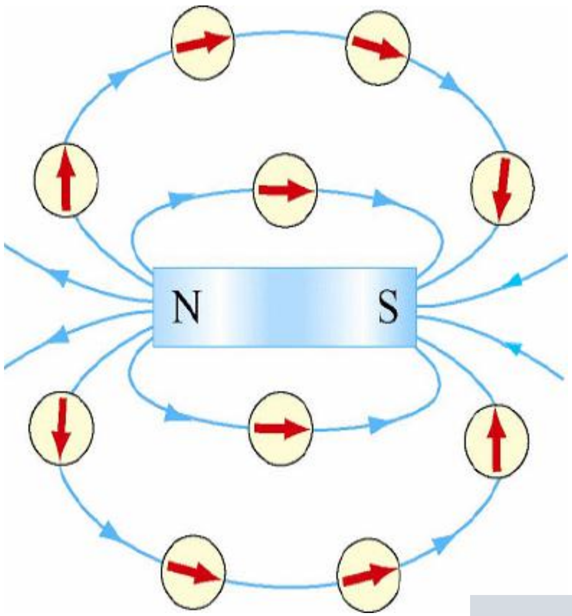
3- Microphone

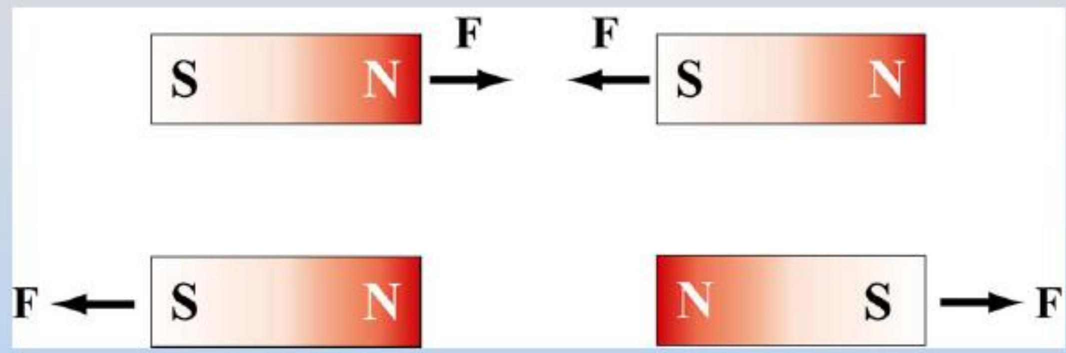
4- Telephone bell rings

5- Television control focusing

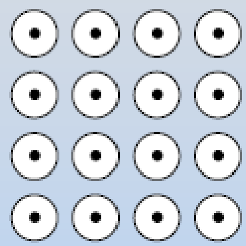
6- Memory store

7- Magnetically levitated high-speed vwhicles





Like poles repel, opposite poles attract



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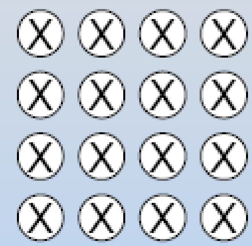

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7-3: Magnetic Force:

- The electric field (**E**) at a point in space has been defined as the electric force ($\mathbf{F_e}$) per unit charge acting on test charge when placed at that point .

$$\vec{\mathbf{F}}_e = q\,\vec{\mathbf{E}}$$

- We now define the magnetic flux density **(B)** at a point in space in terms of the magnetic force **(** $\mathbf{F_m}$ **)** that would exerted on a charged particle moving with a velocity **(u)** were it to be passing through that point.

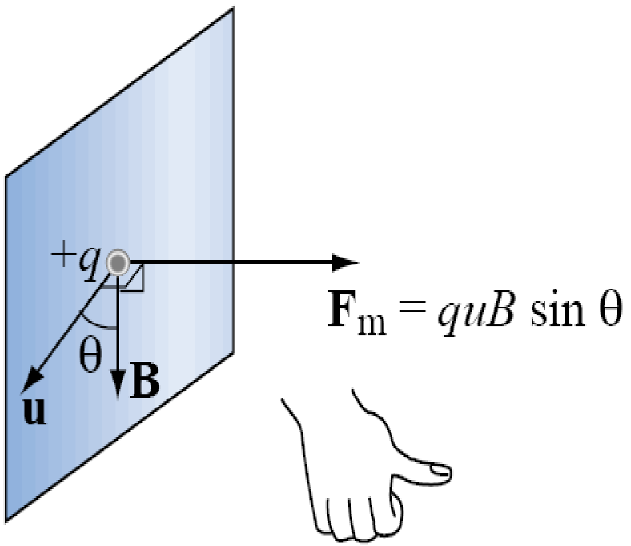
$$\vec{\mathbf{F}}_m = q\,\vec{\mathbf{u}} \times \vec{\mathbf{B}} \qquad where \qquad \mathbf{B} \;\; is \;\; measure \;\; in\; unit \;\; of \;\; (N\,s/C\,m) = N/Am = Tesla$$

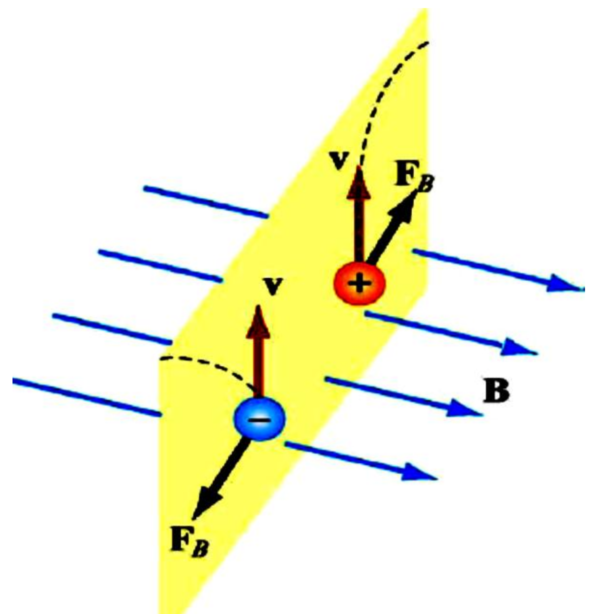
$$and \qquad 1\,T = 10^4 \;\; Gauss$$

$$\mathbf{F}_m = q\,\mathbf{u}\,\mathbf{B}\sin\theta \qquad where \;\; \theta \;\; is \; the \;\; angle \; between \;\; \mathbf{u} \; and \; \mathbf{B}$$

- **We note that (F_m) is maximum when (u) is perpendicular to (B) ($\theta = 90^\circ$), and it is zero when (u) is parallel to (B) ($\theta = 0^\circ, 180^\circ$).**

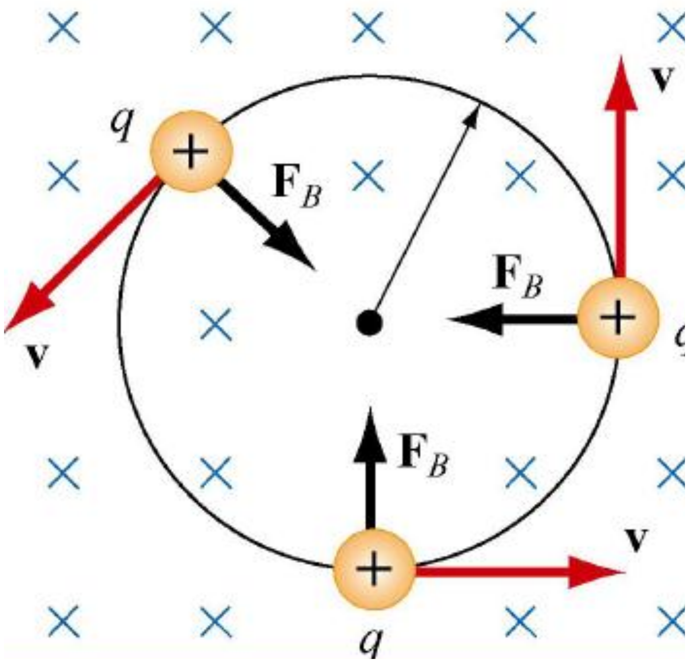
- **For a positively charged particle, the direction of (F_m) is in the direction of cross product ($\vec{u} \times \vec{B}$) , which is perpendicular to the plane containing (u and B) and governed by the right hand rule. If (q) is negative, the direction of (F_m)is reversed as illustrated below. The magnitude of (F_m) is given by:**



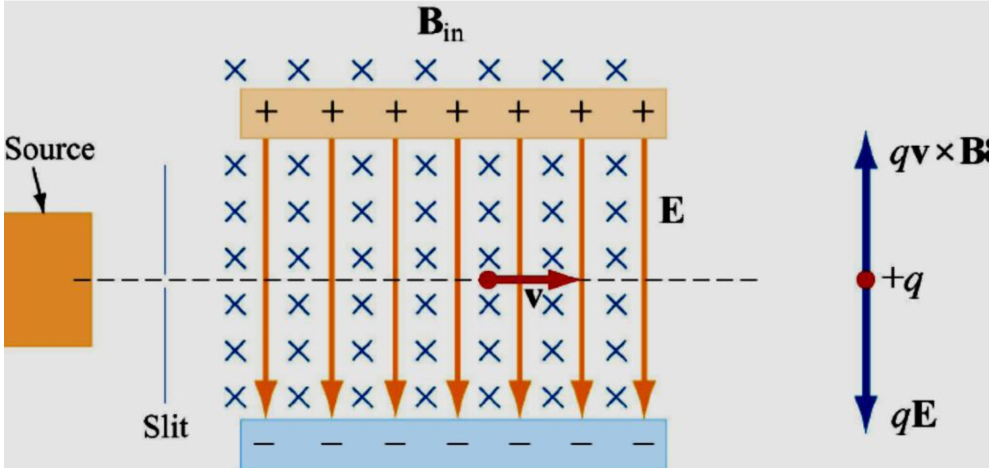


When a charge particle moving with a constant velocity **(u)** and entered to a region contain a constant uniform magnetic field is experienced by a force due to the magnetic field and make it to move in a circular motion as shown below:

If a charged particle is in the presence of both an electric field **(E)** and a magnetic field **(B)**, then the total electromagnetic force acting on it is:



$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_e + \vec{\mathbf{F}}_m = q(\vec{\mathbf{E}} + \vec{\mathbf{u}} \times \vec{\mathbf{B}})$$



Particle moves in a straight line when

$$\vec{\mathbf{F}}_{net} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}) = 0 \Rightarrow \quad v = \frac{E}{B}$$

Fundamental Differences between electric & Magnetic Forces

1- Whereas the electric force (F_e) is always in the direction of the electric field (E), the magnetic force (F_m) is always perpendicular to the magnetic field (B).

2- Whereas the electric force (F_e) acts on a charged particle whether or not it is moving, the magnetic force (F_m) acts on it only when it is in motion.

3-Whereas the electric force (F_e) expends energy in displacing a charged particle the magnetic force (F_m) does no work when a particle is displaced.

7-3-1: Magnetic Force on a Current Carrying Conductor:

A current flowing through a conducting wire consists of charged particles drifting through the material of the wire. Consequently, when a current-carrying wire is placed in a magnetic field, it will experiences a force equal to the sum of the magnetic forces acting on the charged particles moving within it. Therefore, the magnetic force for a current-carrying wire can be expressed as:

$$d\vec{\mathbf{F}}_m = dq\,\vec{\mathbf{u}} \times \vec{\mathbf{B}} \qquad and \qquad dq = \rho_v\, dv$$

$$where \quad \rho_v = N\,e \quad and \quad dv = A\,dl$$

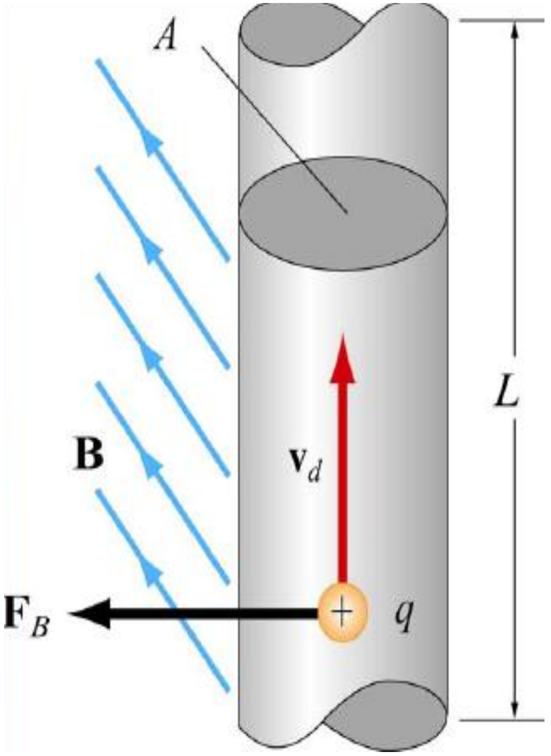
$$d\vec{\mathbf{F}} = N\,e\,A\,u_d\, d\vec{l} \times \vec{\mathbf{B}}$$

$$where \quad \vec{\mathbf{J}} = \rho_v\, u_d = N\,e\,u_d$$

$$and \quad I = J\,A$$

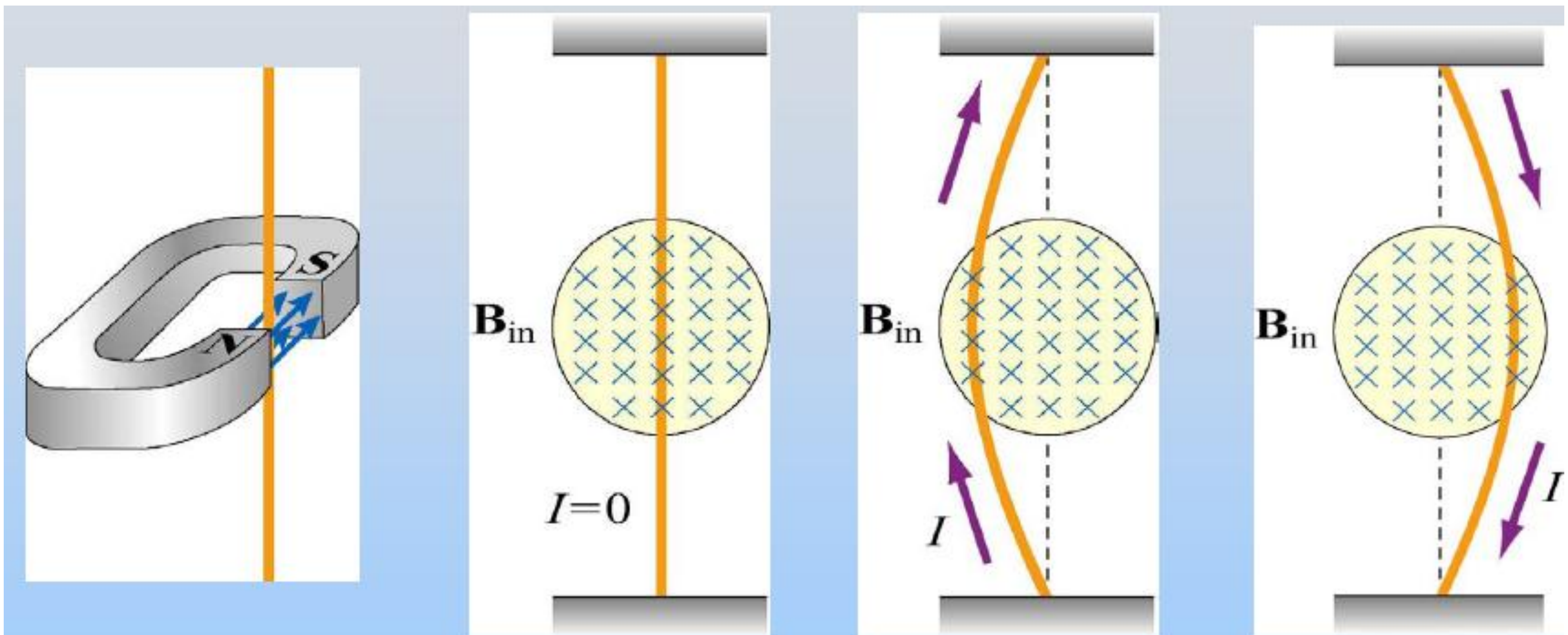
$$\therefore \quad d\vec{\mathbf{F}} = I\, d\vec{l} \times \vec{\mathbf{B}}$$

$$\frac{d\vec{\mathbf{F}}_m}{dq} = \vec{\mathbf{u}} \times \vec{\mathbf{B}} = \vec{\mathbf{E}}$$



Therefore, for a regular segment of conducting wire of length **(L)** and carrying current **(I)**، the magnetic force can be expressed as:

$$\vec{\mathbf{F}}_m = I\,\vec{\mathbf{L}} \times \vec{\mathbf{B}} \Rightarrow\Rightarrow \mathbf{F}_m = I\,L\,\mathbf{B}\,\sin\theta$$



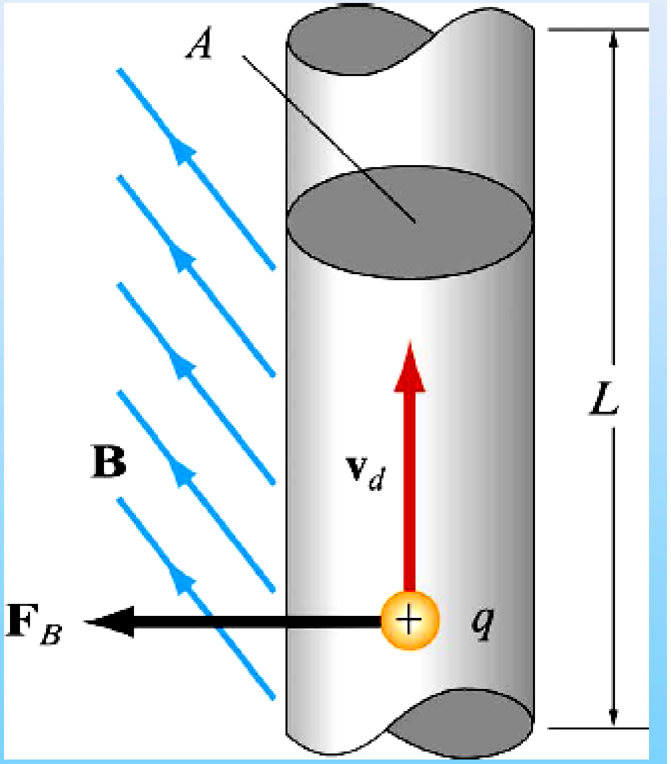
Current is moving charges, and we know that moving charges **feel** a force in a magnetic field

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

$$= (\text{charge})\frac{\text{m}}{\text{s}} \times \vec{\mathbf{B}}$$

$$= \frac{\text{charge}}{\text{s}} \text{m} \times \vec{\mathbf{B}}$$

$$\boxed{\vec{\mathbf{F}}_B = I\left(\vec{\mathbf{L}} \times \vec{\mathbf{B}}\right)}$$



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Examples

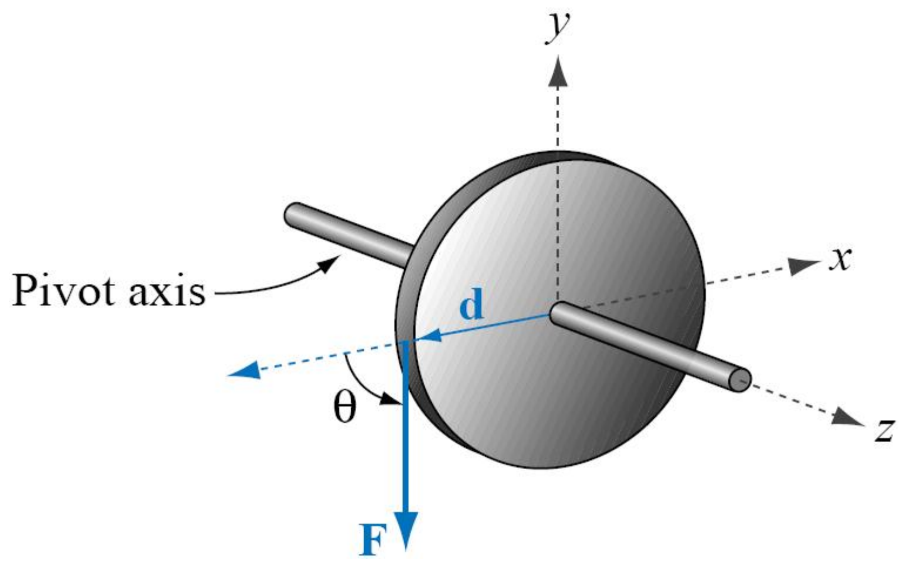
A $(10\ nC)$ **charge particle has a velocity** $\vec{v} = 3\,\hat{a}_x + 4\hat{a}_y + 5\hat{a}_z \quad (m/s)$ **as it enters a magnetic field** $\vec{B} = 1000\ \hat{a}_y \quad (T)$**. Calculate the force vector on the charge and what electric field is required so that the velocity of the charged particle remains constant.**

7-3-2: Magnetic Torque on a Current –Carrying Loop

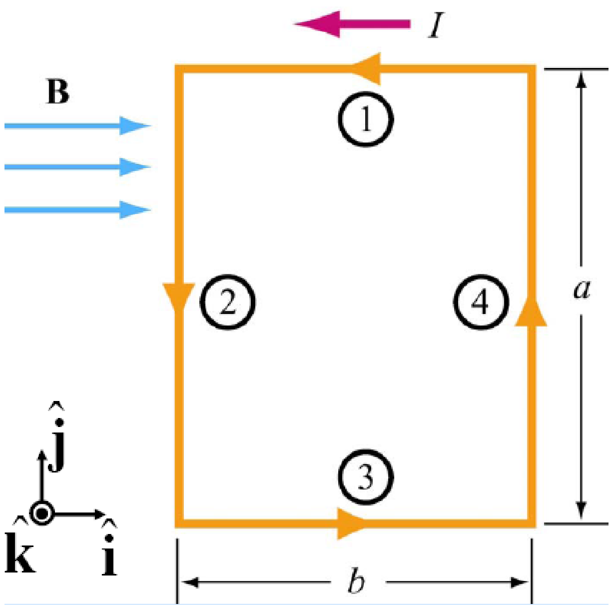
When a force is applied on a rigid body pivoted about a fixed axis, the body will react by rotating about that axis. The strength of the reaction depends on the cross product of the applied force vector (**F)** and the distance vector (**d)** ، measured from a point on the rotation axis (such that **d** is perpendicular to the axis) to the point of a application **(F)** ، see figure below:

The length of **(d)** is called the **moment arm** and the cross product of **(d)** with **(F)** is called the **Torque:**

$$\vec{\mathbf{T}} = \vec{\mathbf{d}} \times \vec{\mathbf{F}}$$



To demonstrate the Torque which produces by a magnetic force, consider a rectangular loop of rigid wires carrying current **(I)** and has a dimensions **(a)** and **(b)** placed in a constant uniform magnetic field **(B)** as shown below:



According to the equation:

$$\vec{\mathbf{F}}_m = I\,\vec{\mathbf{L}} \times \vec{\mathbf{B}} \Rightarrow\Rightarrow \mathbf{F}_m = I\,L\,\mathbf{B}\ \sin\theta$$

$$\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_3 = 0\ \ (\mathbf{I}\,\vec{\mathbf{L}} \parallel \vec{\mathbf{B}})$$

$$\vec{\mathbf{F}}_2 = \mathbf{I}(-a\,\hat{a}_y) \times \mathbf{B}\,\hat{a}_x = \mathbf{I}\,a\,\mathbf{B}\,\hat{a}_z$$

$$\vec{\mathbf{F}}_4 = \mathbf{I}(a\,\hat{a}_y) \times \mathbf{B}\,\hat{a}_x = -\mathbf{I}\,a\,\mathbf{B}\,\hat{a}_z$$

$$\vec{\mathbf{F}}_T = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 + \vec{\mathbf{F}}_4 = 0$$

$$No\ net\ force\ on\ the\ loop$$

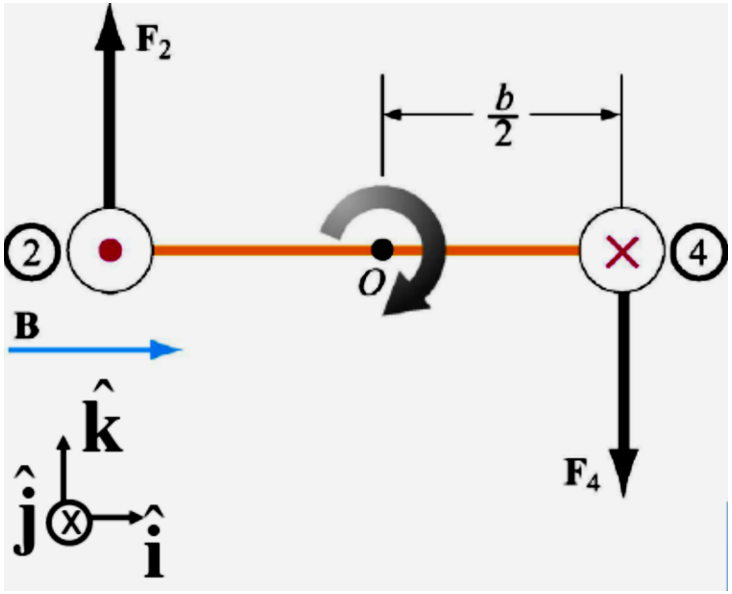
Torque on rectangular loop

$$\vec{\mathbf{T}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$$

$$\vec{\mathbf{T}} = \left(-\frac{b}{2}\ \hat{a}_x \right) \times \vec{\mathbf{F}}_2 + \left(\frac{b}{2}\ \hat{a}_x \right) \times \vec{\mathbf{F}}_4$$

$$\vec{\mathbf{T}} = \left(-\frac{b}{2}\ \hat{a}_x \right) \times (\mathbf{IaB}\ \hat{a}_z) + \left(\frac{b}{2}\ \hat{a}_x \right) \times (-\mathbf{IaB}\ \hat{a}_z)$$

$$\vec{\mathbf{T}} = \frac{\mathbf{Ia\,bB}}{2}\ \hat{a}_y + \frac{\mathbf{Ia\,bB}}{2}\ \hat{a}_y = \mathbf{IabB}\ \hat{a}_y$$



Therefore, the magnetic torque on this loop can be expressed as:

$$\vec{\mathbf{T}} = \mathbf{IAB}\ \hat{a}_y \qquad \vec{\mathbf{A}} = \mathbf{A}\hat{a}_n = \mathbf{a\ b}\ \hat{a}_n \quad \textit{area vector of the loop}$$

$$\text{since}: \ \vec{\mathbf{B}} = \mathbf{B}\ \hat{a}_x \qquad \textit{hence};\ \hat{a}_n = \hat{a}_z \ \textit{ in this example}$$

$$\vec{\boldsymbol{\tau}} = I\vec{\mathbf{A}} \times \vec{\mathbf{B}}$$

Familiar? No net force but there is a torque

Magnetic Dipole Moment

Magnetic dipole moment of a current loop is defined as the product of the current through the loop and the area of the loop, directed normal to the plane of the current loop

$$\vec{\mathbf{m}} = \mathbf{I}\,\mathbf{A}\,\hat{a}_n = \mathbf{I}\,\vec{\mathbf{A}}$$

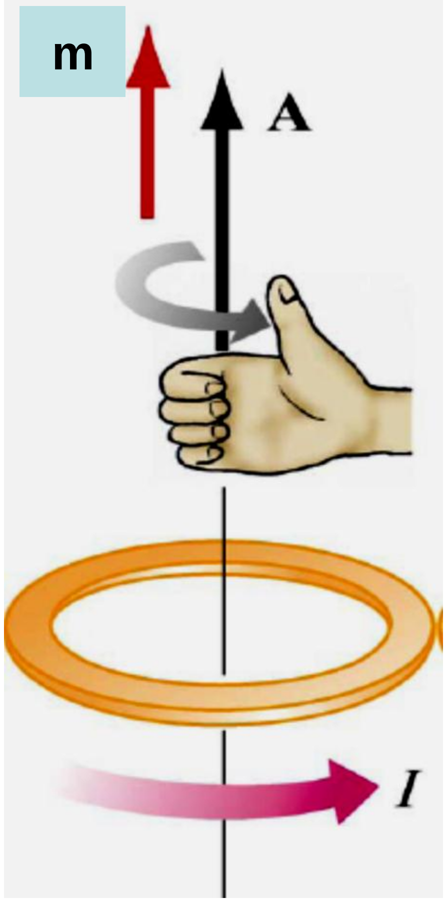
for a loop with N – turns

$$\vec{\mathbf{m}} = N\,I\,\vec{\mathbf{A}}$$

$$Then: \quad \vec{\mathbf{T}} = \vec{\mathbf{m}} \times \vec{\mathbf{B}}$$

$$analoge \quad to \quad \vec{\mathbf{T}} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$$

$\mathbf{T}$ *tends to align* $\mathbf{m}$ *with* $\mathbf{B}$



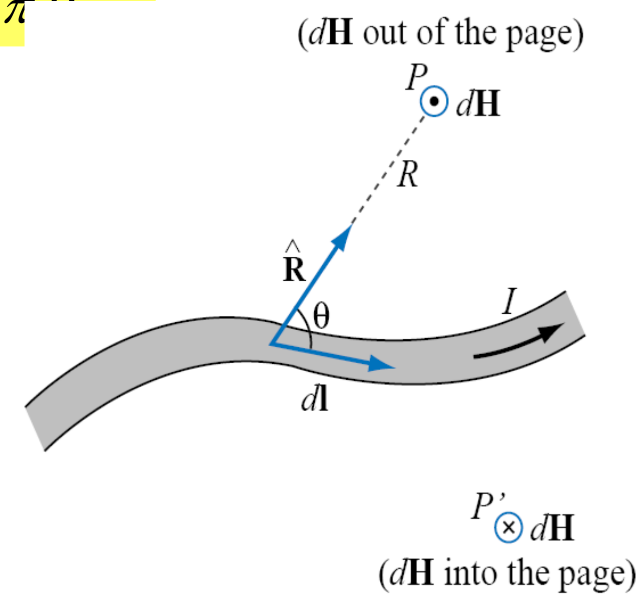
Examples

A circular current loop of radius (r) and current (I) $lies\ \ in\ \ the\ \ z=0-plane$. Find the torque which results if the current is in the $\hat{a}_{\phi}-direction$ and there is a uniform magnetic field $\vec{B}=\frac{B_{\circ}}{\sqrt{2}}(\hat{a}_x+\hat{a}_z)$. $Ans.\ \ \frac{\pi r^2 B_{\circ} I}{\sqrt{2}}\ \hat{a}_y$

7-4: Biot-Savart's Law:

The **Biot-Savart**'s law, states that] The differential magnetic field intensity ***(dH)*** produced at a point (**P)** by the differential current element (**I dl)** is directly proportional to the product of (**I dl)** and the sine of the angle between the current element and the line joining (**P)** to the element and is inversely proportional to the square of the distance (**R)** between (**P)** and the element. That is:

$$d\vec{\mathbf{H}} = \frac{i\,d\vec{l} \times \hat{a}_R}{4\pi R^2} \Rightarrow\Rightarrow where \qquad \mathbf{B} = \mu\,\mathbf{H} \qquad \Rightarrow\Rightarrow d\vec{\mathbf{B}} = \frac{\mu_\circ\, i\,d\vec{l} \times \hat{a}_R}{4\pi R^2}$$



$$i\,d\vec{l} = is\ called\ \ filamentary\,current\ \ (A.m)$$

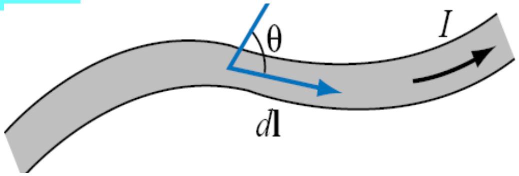
$$R = Distance\ between\ \ (i\,dl)\ and\ \ point\ \mathbf{P}$$

$$\hat{a}_R = Unit\ vector = \frac{\vec{\mathbf{R}}}{|\vec{\mathbf{R}}|}$$

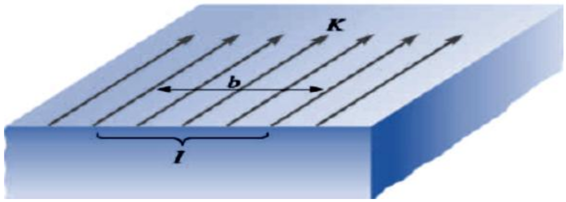
$$\mu_\circ = Magnetic\ permeability\ of\ \ free\,space = 4\pi \times 10^{-7}\ \ (H/m)$$

There are three types of current configurations, they are

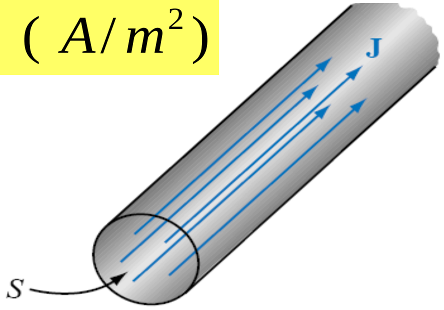
1. Line current density (filamentary Current) $(i\,d\vec{l}\,)$



2. Surface Current Density $(\vec{\mathbf{K}}\,ds)$ $\mathbf{K}$ *measure in unit of* (A/m)



3. Volume Current density $(\vec{\mathbf{J}}\,dv)$ $\mathbf{J}$ *measure in unit of* (A/m^2)



$$i\,dl = k\,ds = J\,dv$$

$$and \quad hence \quad \vec{\mathbf{B}} = \frac{\mu_\circ\, i\, d\vec{l} \times \hat{a}_R}{4\pi \mathbf{R}^2} = \frac{\mu_\circ\, \vec{K}\, ds \times \hat{a}_R}{4\pi \mathbf{R}^2} = \frac{\mu_\circ\, \vec{\mathbf{J}} \times \hat{a}_R\, dv}{4\pi \mathbf{R}^2}$$

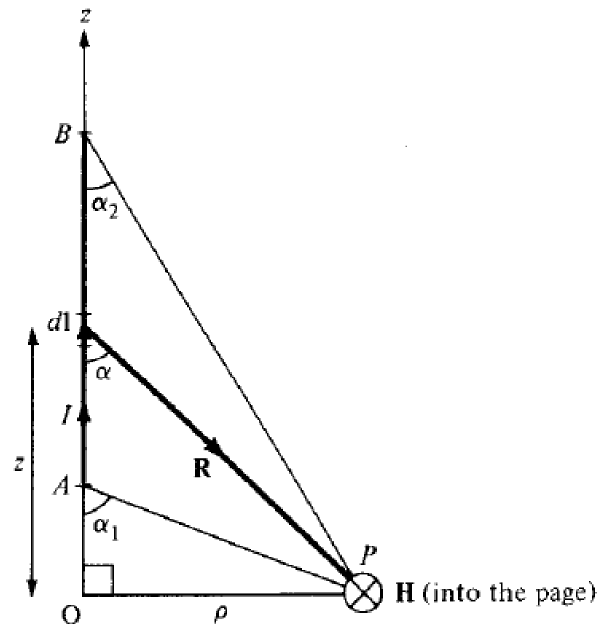
Therefore, the magnetic field for each of the current configurations can be integrally expressed as:

$$\vec{\mathbf{B}} = \frac{\mu_\circ}{4\pi} \int \frac{i\, d\vec{l} \times \hat{a}_R}{\mathbf{R}^2} \qquad for \quad filamentary\ current\ distribution$$

$$\vec{\mathbf{B}} = \frac{\mu_\circ}{4\pi} \int \frac{\vec{K} \times \hat{a}_R}{\mathbf{R}^2}\, ds \qquad for \quad surface\ current\ distribution$$

$$\vec{\mathbf{B}} = \frac{\mu_\circ}{4\pi} \int \frac{\vec{\mathbf{J}} \times \hat{a}_R}{\mathbf{R}^2}\, dv \qquad for\ volume\ current\ distribution$$

Example (1): **A linear conductor of length (L) and carrying a current (I) is placed along the Z-axis as shown in figure below. Determine the magnetic flux density (B) at a point (P) located at a distance (ρ) in the (x-y)-plane in free space.**



$$\vec{\mathbf{B}} = \frac{\mu_\circ}{4\pi} \int \frac{i\, d\vec{l} \times \hat{a}_R}{\mathbf{R}^2}$$

$$i\, d\vec{l} = i\, dz\, \hat{a}_z \qquad \vec{\mathbf{R}} = \rho\, \hat{a}_\rho - z\, \hat{a}_z \qquad \hat{a}_R = \frac{\vec{\mathbf{R}}}{|\vec{\mathbf{R}}|} = \frac{\rho\, \hat{a}_\rho - z\, \hat{a}_z}{\sqrt{\rho^2 + z^2}}$$

$$i\, d\vec{l} \times \hat{a}_R = i\, dz\, \hat{a}_z \times \frac{\rho\, \hat{a}_\rho - z\, \hat{a}_z}{\sqrt{\rho^2 + z^2}} = \frac{i\, \rho\, dz}{\sqrt{\rho^2 + z^2}}\ \hat{a}_\phi$$

$$\vec{\mathbf{B}} = \frac{\mu_\circ}{4\pi}\int \frac{i\,\rho\;dz}{\left(\rho^2 + z^2\right)^{3/2}}\,\hat{a}_\phi = \frac{\mu_\circ\;i\,\rho}{4\pi}\int \frac{dz}{\left(\rho^2 + z^2\right)^{3/2}}\,\hat{a}_\phi$$

$$z = \rho\cot an\,\alpha \qquad\qquad then \quad dz = \rho\cos ec^2\alpha\;d\alpha$$

$$and \;\; hence, \quad \rho^2 + z^2 = \rho^2\,(1 + \cot an^2\alpha) = \rho^2 co\sec^2\alpha$$

$$\vec{\mathbf{B}} = \frac{\mu_\circ\;i\,\rho}{4\pi}\int \frac{\rho\,co\sec^2\alpha\;d\alpha}{\left(\rho^2\cos ec^2\alpha\right)^{3/2}}\,\hat{a}_\phi$$

$$\Rightarrow\Rightarrow \frac{\mu_\circ\;i\,\rho}{4\pi}\int \frac{\rho\,co\sec^2\alpha\;d\alpha}{\rho^3\cos ec^3\alpha}\,\hat{a}_\phi$$

$$= \frac{\mu_\circ\;i}{4\pi\,\rho}\int \sin\alpha\;d\alpha\;\hat{a}_\phi$$

$$\vec{\mathbf{B}} = \frac{\mu_\circ\, i}{4\pi\,\rho}\int \sin\theta\; d\theta\; \hat{a}_\phi = \frac{\mu_\circ\, i}{4\pi\,\rho}\,\hat{a}_\phi\;(\cos\theta)$$

$$= \frac{\mu_\circ\, i}{4\pi\,\rho}\,\hat{a}_\phi\,\left(\frac{z}{\sqrt{\rho^2 + z^2}}\right)\Big|_{-l/2}^{l/2}$$

$$= \frac{\mu_\circ\, i}{4\pi\,\rho}\,\hat{a}_\phi\,\left(\frac{l/2}{\sqrt{\rho^2 + l^2/4}} - \frac{-l/2}{\sqrt{\rho^2 + l^2/4}}\right)$$

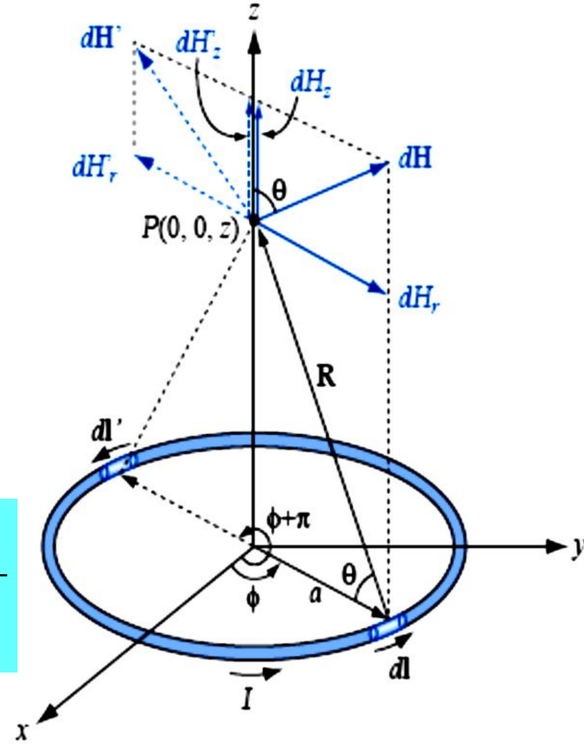
$$\vec{\mathbf{B}} = \frac{\mu_\circ\, i\, l}{2\pi\,\rho}\;\frac{\hat{a}_\phi}{\sqrt{4\rho^2 + l^2}}$$

For an infinitely long wire such that $l \geq\geq \rho$ **, this equation reduces to:**

$$\vec{\mathbf{B}} = \frac{\mu_\circ\, i}{2\pi\,\rho}\,\hat{a}_\phi = \frac{\mu_\circ\, i}{2\pi\, r}\,\hat{a}_\phi = \frac{\mu_\circ\, i}{2\pi\, d}\,\hat{a}_\phi \quad ,$$

Where, **(r) , (d) and (ρ)** are the distance from the wire carrying current and the point (P) at which magnetic field is measured.

Example (2): **A circular loop of radius (ρ) placed on the ($Z = zero$)-plane and carries a steady current (I). Determine the magnetic flux density (B) at a point on the axis of the loop.**



$$\vec{\mathbf{B}} = \frac{\mu_\circ}{4\pi}\int \frac{i\,d\vec{l} \times \hat{a}_R}{\mathbf{R}^2}$$

$$i\,d\vec{l} = i\,\rho\,d\phi\,\hat{a}_\phi \qquad \vec{\mathbf{R}} = -\rho\,\hat{a}_\rho + z\,\hat{a}_z \qquad \hat{a}_R = \frac{\vec{\mathbf{R}}}{|\vec{\mathbf{R}}|} = \frac{-\rho\,\hat{a}_\rho + z\,\hat{a}_z}{\sqrt{\rho^2 + z^2}}$$

$$i\,d\vec{l} \times \hat{a}_R = i\,\rho\,d\phi\,\hat{a}_\phi \times \frac{-\rho\,\hat{a}_\rho + z\,\hat{a}_z}{\sqrt{\rho^2 + z^2}} = \frac{i\,\rho(\rho\,\hat{a}_z + z\,\hat{a}_\rho)\,d\varphi}{\sqrt{\rho^2 + z^2}}$$

But, due to symmetry about $\rho - coordinate$ the $\hat{a}_\rho$ components of magnetic field are vanishes or cancel each other .

$$\vec{\mathbf{B}} = \frac{\mu_\circ}{4\pi} \int \frac{i\,\rho^2\;d\phi}{\left(\rho^2 + z^2\right)^{3/2}}\,\hat{a}_z = \frac{\mu_\circ\;i\,\rho^2}{4\pi}\,\frac{2\pi}{\left(\rho^2 + z^2\right)^{3/2}}\,\hat{a}_z$$

$$\vec{\mathbf{B}} = \frac{\mu_\circ\;i\,\rho^2}{2\left(\rho^2 + z^2\right)^{3/2}}\;\hat{a}_z$$

Therefore, the magnetic flux density of a circular loop at the center of the loop $z = 0$, is given by:

$$\vec{\mathbf{B}} = \frac{\mu_\circ\;i}{2\;\rho}\;\hat{a}_z \quad and \quad for\;\; N - turn\;loop\;\; \vec{\mathbf{B}} = \frac{\mu_\circ\,N\;i}{2\;\rho}\;\hat{a}_z$$

7-5 : Magnetic Force between Two Parallel Conductors

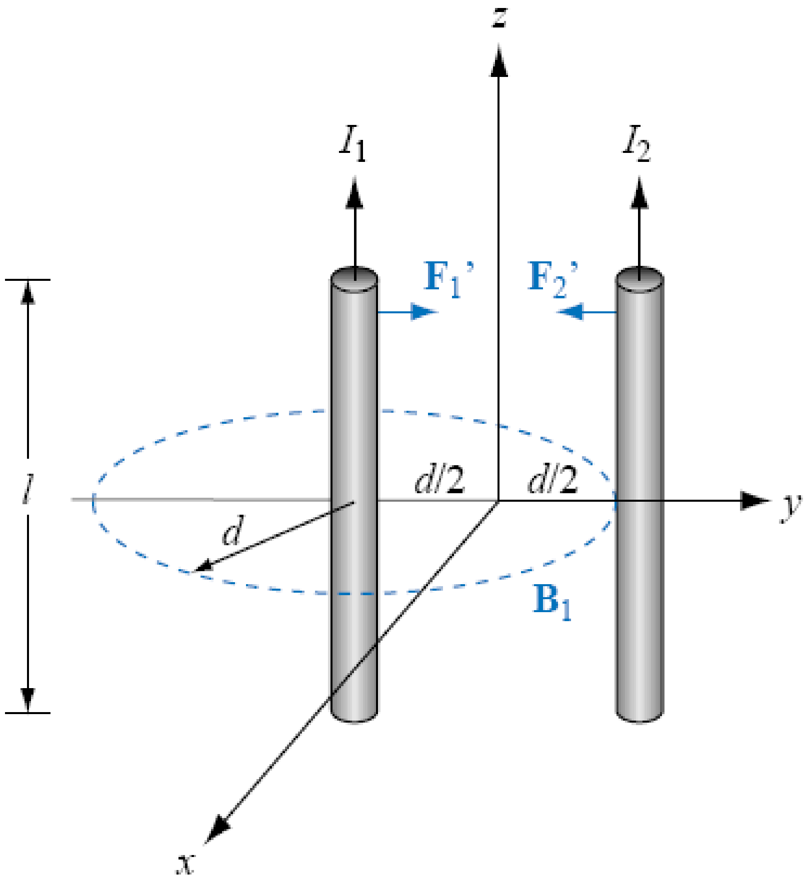
Let us consider two very long (or effectively infinitely long), straight , parallel wires in free space, separated by a distance **(d)** ، and carrying currents (**I_1) and (I_2)** in the same direction, as shown in figure below.

Current (I_1) is located at $y=-d/2$ **, and current (I_2) is located at** $y=d/2$ **and both point in the z-direction.**

$$\vec{\mathbf{B}}=\frac{\mu_\circ I}{2\pi r}\hat{a}_\phi$$

For the wire currying current (I_1) ،the magnetic field at distance (d) is given by:

$$\vec{\mathbf{B}}_1=\frac{\mu_\circ I_1}{2\pi d}(-\hat{a}_x)---------(1)$$



Then the magnetic force (F_2) exerted on a length (L) of wire carrying current (I_2) due to its presence in field (B_1) may be obtained by applying the following equation:

$$\vec{\mathbf{F}}_2 = I_2\, l\, \hat{a}_z \times \vec{\mathbf{B}}_1 \quad \Rightarrow\Rightarrow I_2\, l\, \hat{a}_z \times \frac{\mu_\circ I_1}{2\pi d}(-\hat{a}_x)$$

$$\vec{\mathbf{F}}_2 = \frac{\mu_\circ I_1 I_2\, l}{2\pi d}(-\hat{a}_y) ---------(2)$$

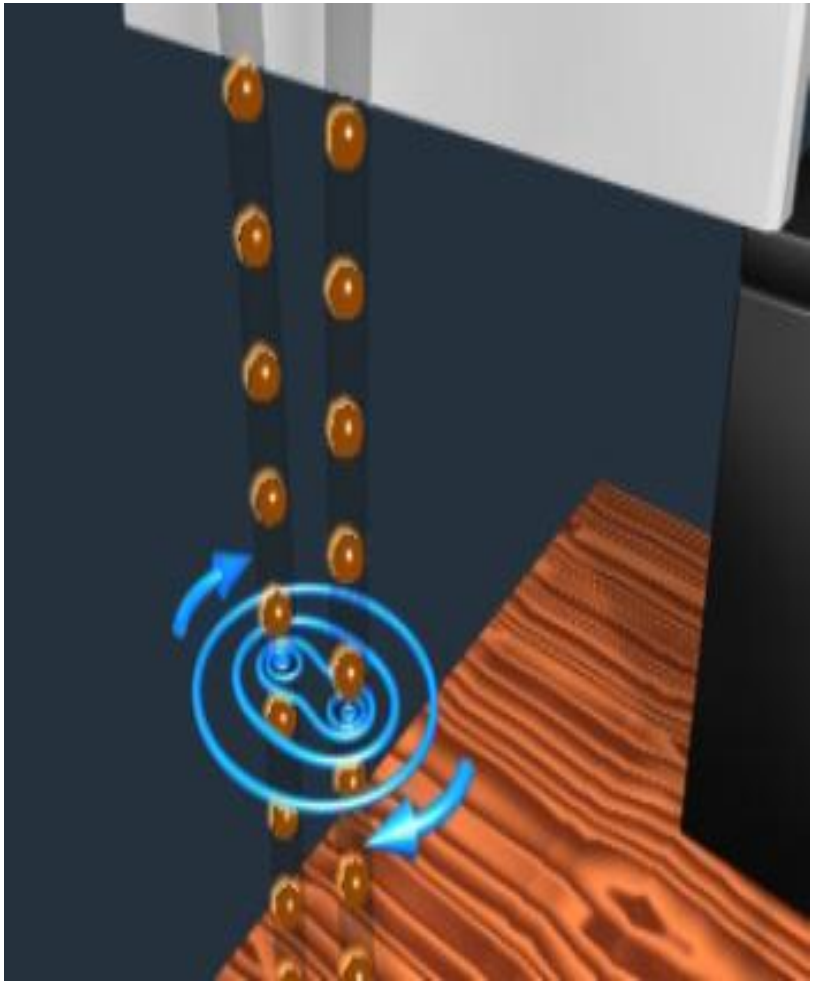
And the corresponding force per unit length is:

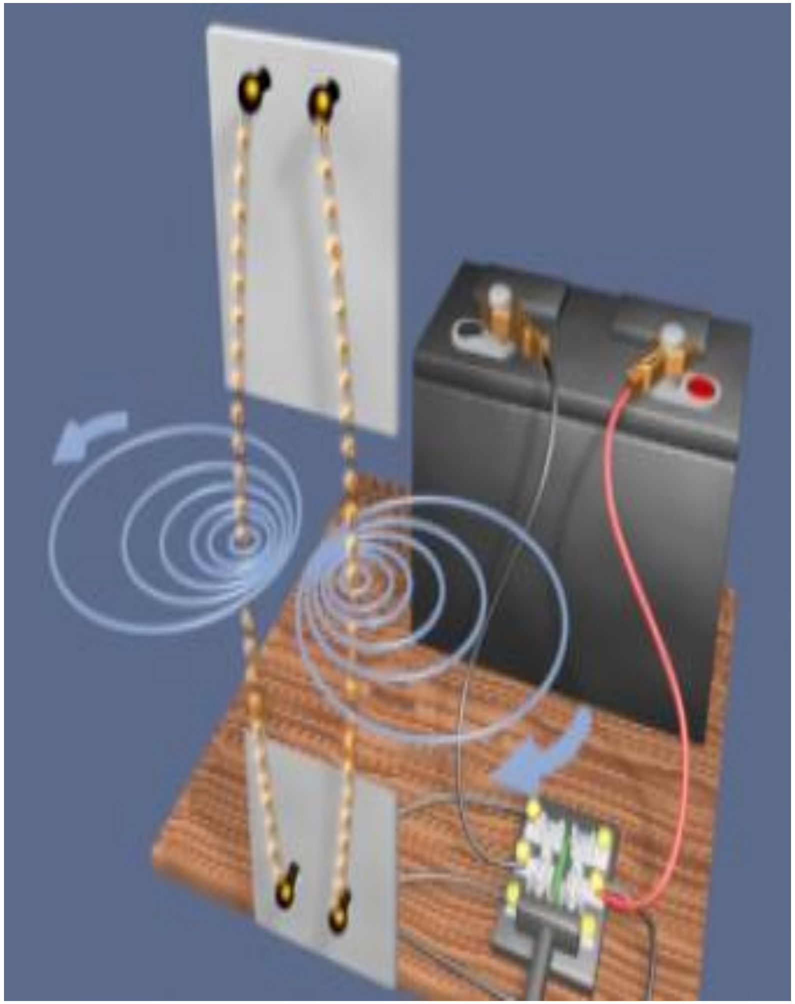
$$\vec{\mathbf{F}}_2' = \frac{\mathbf{F}_2}{l} = \frac{\mu_\circ I_1 I_2}{2\pi d}(-\hat{a}_y) ---------(3)$$

A similar analysis performed for the force per unit length exerted on the wire carrying current (I_1) leads to;

$$\vec{\mathbf{F}}_1' = \frac{\mathbf{F}_1}{l} = \frac{\mu_\circ I_1 I_2}{2\pi d}\hat{a}_y ---------(4)$$

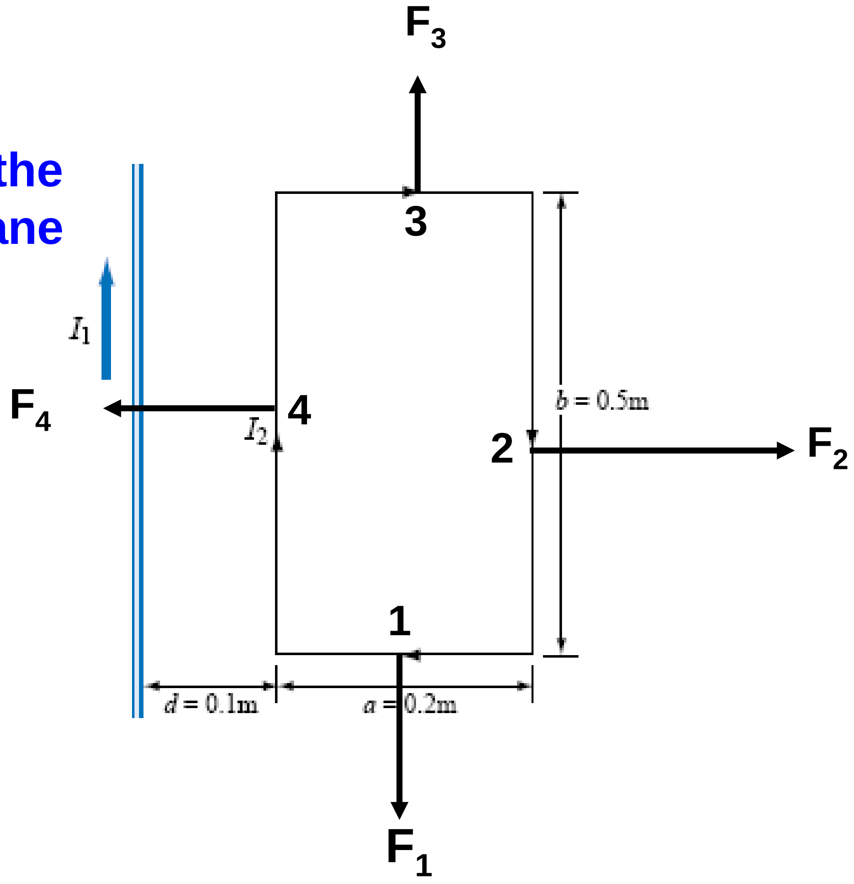
Thus, $F_1 = -F_2$, which means that the two wires attract each other with equal forces. If the currents are in opposite directions, the wires would repel each other with equal forces.





Example(3): The long, straight conductor shown below, lies in the plane of the rectangular loop at a distance (d= 0.1 m). The loop has dimensions (a= 0.2 m) and (b= 0.5 m), and the currents are (I_1= 10 A) and (I_2= 15 A). Determine the net magnetic force acting on the loop.

The magnetic field that produces by the straight wire carrying current (I_1) ، at the plane containing the rectangular loop is given by:



$$\vec{\mathbf{B}}_1 = \frac{\mu_\circ \mathbf{I}_1}{2\pi r}(-\hat{a}_x)$$

Then the magnetic force exerted by this field on the individual dimensions of the loop is calculated as:

$$\vec{\mathbf{F}}_1 = \mathbf{I}_2\, a\,(-\hat{a}_y)\times\frac{\mu_\circ \mathbf{I}_1}{2\pi(d+a/2)}(-\hat{a}_x) = \frac{\mu_\circ\, a\, \mathbf{I}_2 \mathbf{I}_1}{2\pi(d+a/2)}(-\hat{a}_z) ----------(1)$$

$$\vec{\mathbf{F}}_2 = \mathbf{I}_2\, b(-\hat{a}_z)\times\frac{\mu_\circ \mathbf{I}_1}{2\pi(d+a)}(-\hat{a}_x) = \frac{\mu_\circ\, b\, \mathbf{I}_2 \mathbf{I}_1}{2\pi(d+a)}(\hat{a}_y) -------------(2)$$

$$\vec{\mathbf{F}}_3 = \mathbf{I}_2\, a\,(\hat{a}_y)\times\frac{\mu_\circ \mathbf{I}_1}{2\pi(d+a/2)}(-\hat{a}_x) = \frac{\mu_\circ\, a\, \mathbf{I}_2 \mathbf{I}_1}{2\pi(d+a/2)}(\hat{a}_z) ----------(3)$$

$$\vec{\mathbf{F}}_4 = \mathbf{I}_2\, b\,(\hat{a}_z)\times\frac{\mu_\circ \mathbf{I}_1}{2\pi d}(-\hat{a}_x) = \frac{\mu_\circ\, b\, \mathbf{I}_2 \mathbf{I}_1}{2\pi d}(-\hat{a}_y) ----------------(4)$$

Thus, the total force is the sum of these forces and because the forces in the z-direction have the same magnitude and are opposite to each other, they are cancels each other. While the other two force are adding and gives:

$$\vec{\mathbf{F}}_t = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 + \vec{\mathbf{F}}_4 = \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_4 = \frac{\mu_\circ\, b\, \mathbf{I}_1 \mathbf{I}_2}{2\pi}\left(\frac{1}{d+a} - \frac{1}{d}\right)\hat{a}_y$$

$$\vec{\mathbf{F}}_t = \frac{4\pi\times 10^{-7}\times 0.5\times 15\times 10}{2\pi}\left(\frac{1}{0.1+0.2} - \frac{1}{0.1}\right)\hat{a}_y$$

$$\Rightarrow\Rightarrow \vec{\mathbf{F}}_t = 1.5\times 10^{-5}(-6.66)\,\hat{a}_y = -0.1\,\hat{a}_y \quad (mN)$$