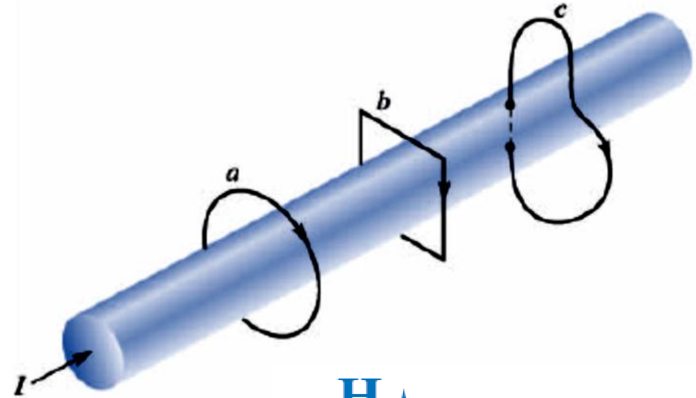
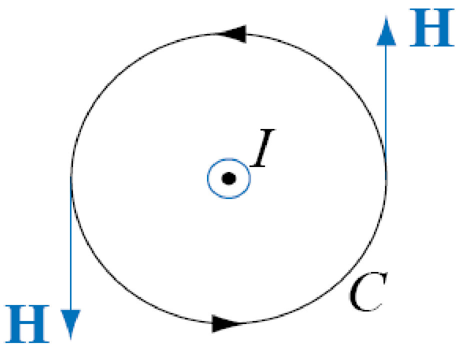
7-7 : Ampere's Law

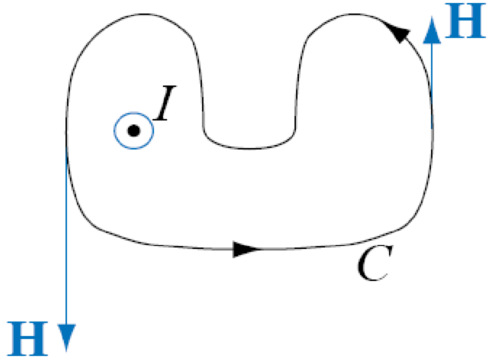
The line integral of the tangential component of (H)around any closed path is the same as the net current (I_{net}) enclosed by the path and mathematically expressed as:

$$\oint_l \vec{\mathbf{H}} \cdot d\vec{l} = \mathbf{I}_{enc} \quad ---------(1)$$

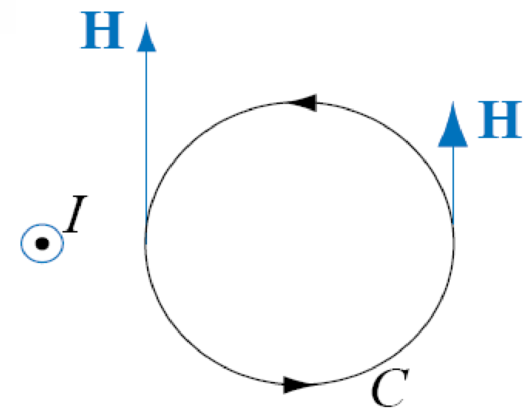




a



b

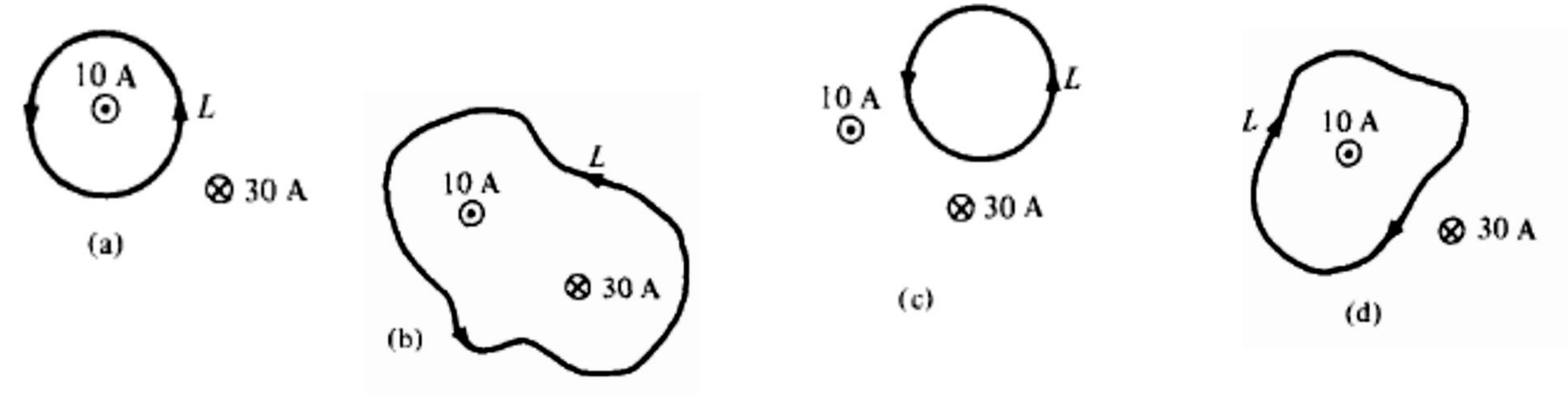


c

According to Ampere's law, equation (1) has the following values for each of the above figures:

$$\oint_c \vec{\mathbf{H}} \cdot d\vec{l} = \mathbf{I}_{enc} \quad for \quad Figure \quad (a) \quad and \quad (b)$$

$$\oint_c \vec{\mathbf{H}} \cdot d\vec{l} = 0 \quad for \quad figure \quad (c)$$



By applying the Stoke's theorem to the left hand side of eq.(1) we obtain:

$$\oint_l \vec{\mathbf{H}}\cdot d\vec{l} = \oint (\vec{\nabla}\times\vec{\mathbf{H}})\cdot d\vec{s} \text{ ---------------(2)}$$

$$\mathbf{I}_{enc} = \oint \vec{\mathbf{J}}\cdot d\vec{s} \text{ ------------------(3)}$$

Substituting eq.(2) and eq.(3) into eq.(1) we get: $\vec{\nabla}\times\vec{\mathbf{H}}=\vec{\mathbf{J}} \text{ ----(4)}$

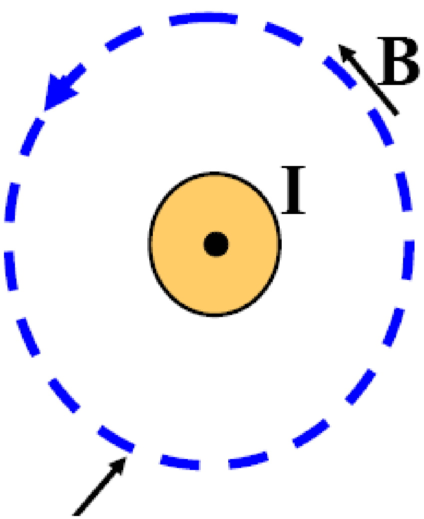
This equation is called 3rd –Maxwell's equation or point form of Ampere's law which indicates that :[The magnetostatic field is a non-conservative field] or [The magnetic field is produced due to a flow of steady current].

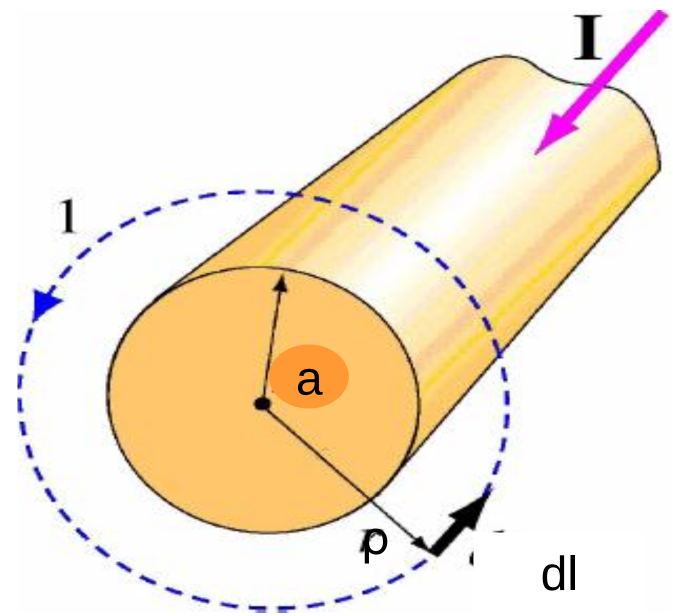
7-7: Application of Ampere's Law

We now applying Ampere's circuit law to determine **(H)** for some symmetrical current distributions as we done for Gauss's law. We will consider an **infinite line current**, an **infinite current sheet** and an **infinity long Coaxial transmission line**.

7-7-1: Infinite Line Current:

Consider an infinitely long filamentary current (**I)** along the z-axis as shown below. To determine (**H)** at an observation point out side and inside the wire ، we allow a closed path passes through both regions **.** These paths on which Ampere's law is to be applied, is known as an **Amperian path [Analogous to term of Gauss's Surface].**





Region 1: Outside wire $\rho \geq a$

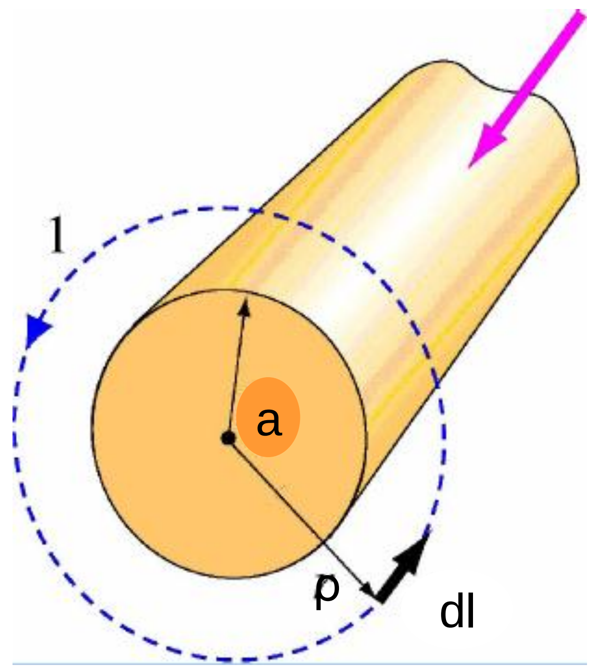
$$\oint_c \vec{\mathbf{H}} \cdot d\vec{l} = \mathbf{I}_{enc} \quad \Rightarrow\Rightarrow\Rightarrow \quad \vec{\mathbf{H}} = \mathbf{H}_\phi \, \hat{a}_\phi \qquad and \qquad d\vec{l} = \rho \, d\phi \, \hat{a}_\phi$$

$$Hence: \quad \oint_c \mathbf{H}_\phi \, \hat{a}_\phi \cdot \rho \, d\phi \, \hat{a}_\phi = \mathbf{I}_{enc}$$

$$\mathbf{I}_{enc} = \int \vec{\mathbf{J}} \cdot \overrightarrow{ds} = \int_0^a \int_0^{2\pi} \frac{\mathbf{I}}{\pi a^2} \hat{a}_z \cdot \rho \, d\rho \, d\phi \, \hat{a}_z = \frac{\mathbf{I}}{\pi a^2} \frac{a^2}{2} (2\pi) = \mathbf{I}$$

$$\therefore \; \mathbf{H}_\phi (2\pi \rho) = \mathbf{I}_{enc} = \mathbf{I} \quad \Rightarrow\Rightarrow \quad \mathbf{H}_\phi = \frac{\mathbf{I}}{2\pi \rho}$$

$$thus: \; \vec{\mathbf{H}} = \frac{\mathbf{I}}{2\pi \rho} \hat{a}_\phi$$



Region 2: Inside wire $\rho < a$

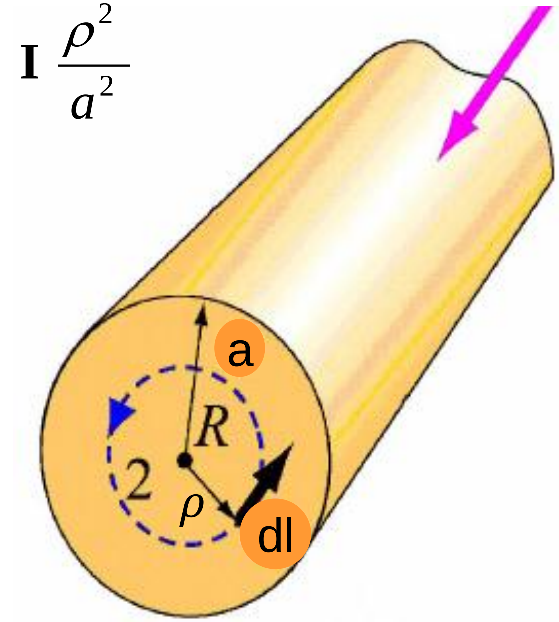
$$\oint_c \vec{\mathbf{H}} \cdot d\vec{l} = \mathbf{I}_{enc} \quad \Rightarrow\Rightarrow\Rightarrow \quad \vec{\mathbf{H}} = \mathbf{H}_\phi \, \hat{a}_\phi \qquad \textit{and} \qquad d\vec{l} = \rho \, d\phi \, \hat{a}_\phi$$

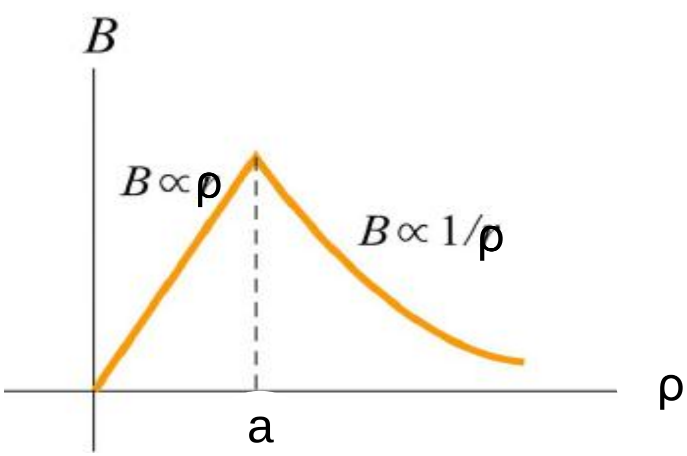
$$\textit{Hence}: \quad \oint_c \mathbf{H}_\phi \, \hat{a}_\phi \cdot \rho \, d\phi \, \hat{a}_\phi = \mathbf{I}_{enc}$$

$$\mathbf{I}_{enc} = \int \vec{\mathbf{J}} \cdot \overrightarrow{ds} = \int_0^{\rho} \int_0^{2\pi} \frac{\mathbf{I}}{\pi a^2} \hat{a}_z \cdot \rho \, d\rho \, d\phi \, \hat{a}_z = \frac{\mathbf{I}}{\pi a^2} \frac{\rho^2}{2} (2\pi) = \mathbf{I} \frac{\rho^2}{a^2}$$

$$\therefore \; \mathbf{H}_\phi (2\pi\rho) = \mathbf{I}_{enc} = \mathbf{I} \frac{\rho^2}{a^2} \quad \Rightarrow\Rightarrow \quad \mathbf{H}_\phi = \frac{\mathbf{I}\,\rho}{2\pi a^2}$$

$$\textit{thus}: \quad \vec{\mathbf{H}} = \frac{\mathbf{I}\,\rho}{2\pi a^2} \hat{a}_\phi$$





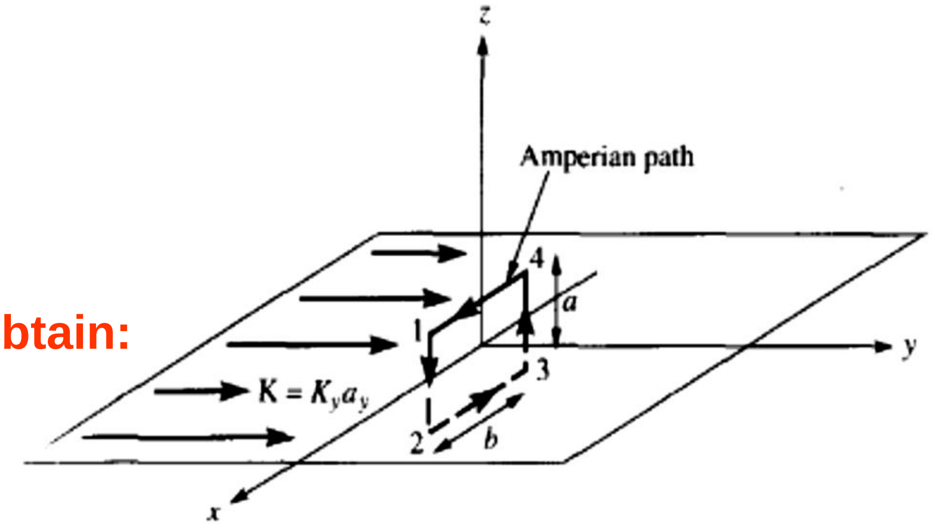
$$\mathbf{B}_{in} = \frac{\mu_\circ \mathbf{I}\,\rho}{2\pi a^2} \qquad \mathbf{B}_{out} = \frac{\mu_\circ \mathbf{I}}{2\pi\rho}$$

7-7-2: Infinite Sheet Current:

We consider an infinite sheet current at the Z= 0 –plane. If the sheet has a uniform current dens $\vec{K} = K_y \hat{a}_y \ (A/m)$. To determine the magnitude of (H), applying Ampere's law by considering a rectangular closed path [Amperian path] of length (b) and width (a), gives:

$$\oint_c \vec{H} \cdot d\vec{l} = I_{enc} = K_y b -------(1)$$

Applying Ampere's law to this path, we obtain:



$$\oint_c \vec{H} \cdot d\vec{l} = [\int_1^2 \vec{H} \cdot d\vec{l} + \int_2^3 \vec{H} \cdot d\vec{l} + \int_3^4 \vec{H} \cdot d\vec{l} + \int_4^1 \vec{H} \cdot d\vec{l} \quad ---(2)$$

$$\mathbf{H} = \begin{cases} H_o \mathbf{a}_x & z > 0 \\ -H_o \mathbf{a}_x & z < 0 \end{cases}$$

$$\oint_c \vec{H} \cdot d\vec{l} = \int_2^3 H_x (-\hat{a}_x) \cdot dx (-\hat{a}_x) + \int_4^1 H_x \hat{a}_x \cdot dx \ \hat{a}_x = K_y \ b \ --------(3)$$

Where the integral $\int_1^2 \vec{\mathbf{H}}\cdot d\vec{l}$ *and* $\int_3^4 \vec{\mathbf{H}}\cdot d\vec{l}$ are zero, since (H) is perpendicular to (dl).

Then eq.(3) reduces to:

$$\oint_c \vec{\mathbf{H}}\cdot d\vec{l} = (-\mathbf{H}_x)(-b) + \mathbf{H}_x b = 2\mathbf{H}_x b = \mathbf{K}_y b$$

$$Hence:\ \mathbf{H}_x = \frac{\mathbf{K}_y}{2}$$

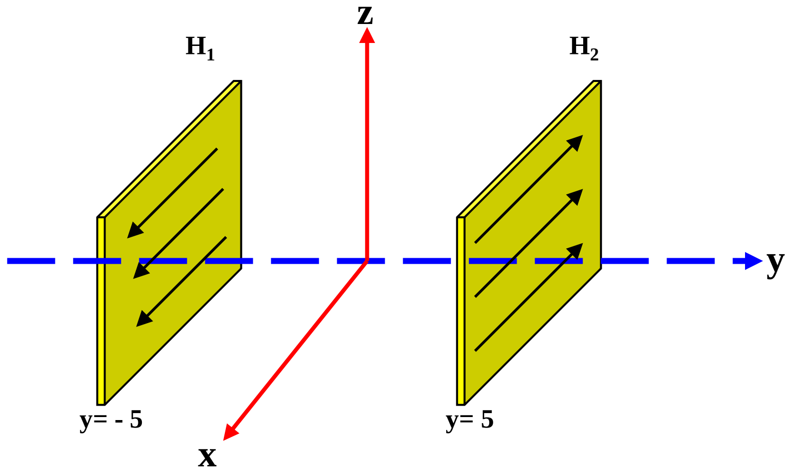
$$Thus:\ \vec{\mathbf{H}} = \frac{1}{2}\vec{\mathbf{K}} \times \hat{a}_n \text{-----------} (4)$$

Where, (a_n) is a unit vector at the surface current and is directed to the region where (H) is being found.

Or: (a_n) is a unit normal vector directed from the current sheet to the point at which the magnetic field intensity is measured.

Example(4): Plane $y = 5m$ carries a sheet current of $\vec{\mathbf{K}} = -20\hat{a}_x \ mA/m$, while plane $y = -5m$, carries a current sheet of $\vec{\mathbf{K}} = 20\hat{a}_x \ mA/m$. Find the magnetic field intensity in all regions surrounding these two sheet currents.

Solution:



(1): Region y ≥ 5

$$\vec{\mathbf{H}} = \frac{1}{2}\vec{\mathbf{K}} \times \hat{a}_n$$

$$\vec{\mathbf{H}}_1 = \frac{1}{2}\vec{\mathbf{K}}_1 \times \hat{a}_y = 10\,\hat{a}_x \times \hat{a}_y = 10\,(A/m)$$

$$\vec{\mathbf{H}}_2 = \frac{1}{2}\vec{\mathbf{K}}_2 \times \hat{a}_y = -10\,\hat{a}_x \times \hat{a}_y = -10\,(A/m)$$

$$\vec{\mathbf{H}}_T = \vec{\mathbf{H}}_1 + \vec{\mathbf{H}}_2 = zero$$

(2): Region y ≤ - 5

$$\vec{\mathbf{H}} = \frac{1}{2}\vec{\mathbf{K}} \times \hat{a}_n$$

$$\vec{\mathbf{H}}_1 = \frac{1}{2}\vec{\mathbf{K}}_1 \times \hat{a}_y = 10\,\hat{a}_x \times (-\hat{a}_y) = -10\,(A/m)$$

$$\vec{\mathbf{H}}_2 = \frac{1}{2}\vec{\mathbf{K}}_2 \times \hat{a}_y = -10\,\hat{a}_x \times (-\hat{a}_y) = 10\,(A/m)$$

$$\vec{\mathbf{H}}_T = \vec{\mathbf{H}}_1 + \vec{\mathbf{H}}_2 = zero$$

(3): Region 5 ≥ y ≥ - 5

$$\vec{\mathbf{H}} = \frac{1}{2}\vec{\mathbf{K}} \times \hat{a}_n$$

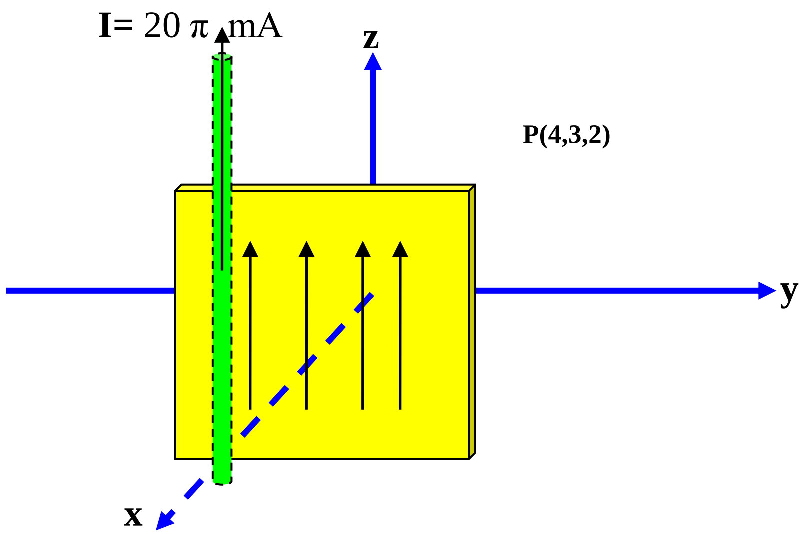
$$\vec{\mathbf{H}}_1 = \frac{1}{2}\vec{\mathbf{K}}_1 \times \hat{a}_y = 10\,\hat{a}_x \times \hat{a}_y = 10\,(A/m)$$

$$\vec{\mathbf{H}}_2 = \frac{1}{2}\vec{\mathbf{K}}_2 \times \hat{a}_y = -10\,\hat{a}_x \times (-\hat{a}_y) = 10\,(A/m)$$

$$\vec{\mathbf{H}}_T = \vec{\mathbf{H}}_1 + \vec{\mathbf{H}}_2 = 10\,\hat{a}_x + 10\,\hat{a}_x = 20\,\hat{a}_x$$

Example(5): Plane $x=10$ carries sheet current($\mathbf{K}=10\ mA/m \quad along \quad z-axis$), while $x=1\ ,\ y=-2$ carries filamentary current ($\mathbf{I}=20\pi\ mA$) along $z-axis$. Determine the magnetic field intensity (H) at $(4,3,2)$.

Solution:



(1): The magnetic field produced at point (4,3,2) due to the sheet current can be calculated as:

$$\vec{\mathbf{H}}_s=\frac{1}{2}\vec{\mathbf{K}}\times\hat{a}_n \quad =5\hat{a}_z\times(-\hat{a}_x)=-5\hat{a}_y\ mA/m \text{------}(1)$$

(2): The magnetic field produced at point (4,3,2) due to the filamentary current can be calculated as:

$$d\vec{\mathbf{H}}=\frac{i\,\vec{dl}\times\hat{a}_R}{4\pi R^2} \qquad and \qquad \vec{\mathbf{R}}=(4-1)\,\hat{a}_x+(3-(-2))\,\hat{a}_y+(2-z)\,\hat{a}_z=3\hat{a}_x+5\hat{a}_y+(2-z)\hat{a}_z$$

$$\hat{a}_R=\frac{\vec{\mathbf{R}}}{|\mathbf{R}|}=\frac{3\hat{a}_x+5\hat{a}_y+(2-z)\hat{a}_z}{\sqrt{9+25+(2-z)^2}}=\frac{3\hat{a}_x+5\hat{a}_y+(2-z)\hat{a}_z}{\sqrt{34+(2-z)^2}} \qquad and \quad i\vec{dl}=20\pi\,dz\ \hat{a}_z$$

$$\therefore\ \vec{\mathbf{H}}_l=\int\frac{20\pi\,dz\ \hat{a}_z\times[3\hat{a}_x+5\hat{a}_y+(2-z)\hat{a}_z]}{4\pi\left(34+(2-z)^2\right)^{3/2}}=\int\frac{(15\hat{a}_y-25\hat{a}_x)}{\left(34+(2-z)^2\right)^{3/2}}dz \text{-----}(2)$$

let: $2 - z = \sqrt{34}\,\tan\theta \qquad \therefore\; dz = -\sqrt{34}\,\sec^2\theta\,d\theta$

Substituting these quantities into eq.(2) and making some algebraic arrangement we get:

$$\therefore \vec{\mathbf{H}}_l = -\int \frac{(15\hat{a}_y - 25\hat{a}_x)\,\sqrt{34}\sec^2\theta\,d\theta}{34\sqrt{34}\,\sec^3\theta} = \frac{(25\hat{a}_x - 15\hat{a}_y)}{34}\int \cos\theta\,d\theta$$

$$\therefore \vec{\mathbf{H}}_l = \frac{(25\hat{a}_x - 15\hat{a}_y)}{34}(\sin\theta) = \frac{(25\hat{a}_x - 15\hat{a}_y)}{34}\left(\frac{2-z}{\sqrt{34+(2-z)^2}}\right)\Bigg|_{-\infty}^{\infty}$$

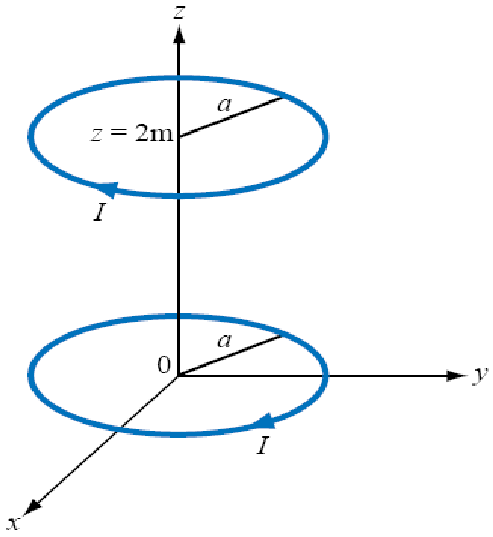
$$\therefore \vec{\mathbf{H}}_l = \frac{(25\hat{a}_x - 15\hat{a}_y)}{34}\times(1-(-1)) = \frac{(25\hat{a}_x - 15\hat{a}_y)}{17} --------(3)$$

adding eqs(1) with (3) we get the total magnetic field intensity:

Example(6): **Two circular filamentary loop are located at** $(r = a \quad , \; z = 0 \;\; and \quad z = 2m \;)$

Find the magnetic field intensity (H) at the midway between the circular loops if the loop wh located at $z = 2m$ **, carries a current of (2 A) in the** $-\hat{a}_\phi - direction$ **and that located at** $z = 0$ **is in the :**

(1)- $\hat{a}_\phi - direction$ **(2)-** $-\hat{a}_\phi - direction$



Solution:

The magnetic field intensity at the midway between the loops due to a circular loop located at (z = 2 m) is calculated as:

$$d\vec{\mathbf{H}}_1 = \frac{i\,\overrightarrow{dl} \times \hat{a}_{R1}}{4\pi \mathbf{R}_1^{\,2}} \qquad \vec{\mathbf{R}}_1 = (0-a)\,\hat{a}_\rho + (1-2)\,\hat{a}_z \qquad \hat{a}_{R1} = \frac{-a\;\hat{a}_\rho - \hat{a}_z}{\sqrt{a^2+1}}$$

$$and \qquad i\;\overrightarrow{dl} = -i\,a\,d\phi\;\hat{a}_\phi$$

$$\therefore\quad d\vec{\mathbf{H}}_1 = \frac{-i\,a\,d\phi\;\hat{a}_\phi \times (-a\;\;\hat{a}_\rho - \hat{a}_z\,)}{4\pi\left(a^2+1\right)^{3/2}} = \frac{i\,a}{4\pi\left(a^2+1\right)^{3/2}}\;(-a\,\hat{a}_z + \hat{a}_\rho)\int_0^{2\pi} d\phi$$

$$\vec{\mathbf{H}}_1 = \frac{-i\,a^2}{2\left(a^2+1\right)^{3/2}}\;\hat{a}_z \quad , \quad due\,to\;\; symmerty\;\; \hat{a}_\rho - components\;\; vanishes$$

(1). The magnetic field intensity at the midway between the loops due to a circular loop located at (z = 0) and current in which is in the $\hat{a}_\phi - direction$ is calculated as:

$$d\vec{\mathbf{H}}_2 = \frac{i\,\overrightarrow{dl} \times \hat{a}_{R2}}{4\pi \mathbf{R}_2^{\,2}} \qquad \vec{\mathbf{R}}_2 = (0-a)\,\hat{a}_\rho + (1-0)\,\hat{a}_z \qquad \hat{a}_{R1} = \frac{-a\ \hat{a}_\rho + \hat{a}_z}{\sqrt{a^2+1}}$$

$$and \quad i\,\overrightarrow{dl} = i\,a\,d\phi\ \hat{a}_\phi$$

$$\therefore\ d\vec{\mathbf{H}}_2 = \frac{i\,a\,d\phi\ \hat{a}_\phi \times (-a\ \hat{a}_\rho + \hat{a}_z)}{4\pi\left(a^2+1\right)^{3/2}} = \frac{i\,a}{4\pi\left(a^2+1\right)^{3/2}}\,(a\,\hat{a}_z - \hat{a}_\rho)\int_0^{2\pi} d\phi$$

$$\vec{\mathbf{H}}_2 = \frac{i\,a^2}{2\left(a^2+1\right)^{3/2}}\ \hat{a}_z \ , \quad due\,to\ symmerty\ \hat{a}_\rho - components\ vanishes$$

$$Hence \quad \vec{\mathbf{H}}_T = \vec{\mathbf{H}}_1 + \vec{\mathbf{H}}_2 = zero$$

(2). The magnetic field intensity at the midway between the loops due to a circular loop located at (z = 0) and current in which is in the $-\hat{a}_\phi - direction$)is calculated as:

$$d\vec{\mathbf{H}}_2 = \frac{i\,\overrightarrow{dl} \times \hat{a}_{R2}}{4\pi \mathbf{R}_2^{\,2}} \qquad \vec{\mathbf{R}}_2 = (0-a)\,\hat{a}_\rho + (1-0)\,\hat{a}_z \qquad \hat{a}_{R1} = \frac{-a\ \hat{a}_\rho + \hat{a}_z}{\sqrt{a^2+1}}$$

$$and \quad i\,\overrightarrow{dl} = -i\,a\,d\phi\ \hat{a}_\phi$$

$$\therefore\ d\vec{\mathbf{H}}_2 = \frac{-i\,a\,d\phi\ \hat{a}_\phi \times (-a\ \hat{a}_\rho + \hat{a}_z)}{4\pi\left(a^2+1\right)^{3/2}} = \frac{i\,a}{4\pi\left(a^2+1\right)^{3/2}}\,(-a\,\hat{a}_z - +\hat{a}_\rho)\int_0^{2\pi} d\phi$$

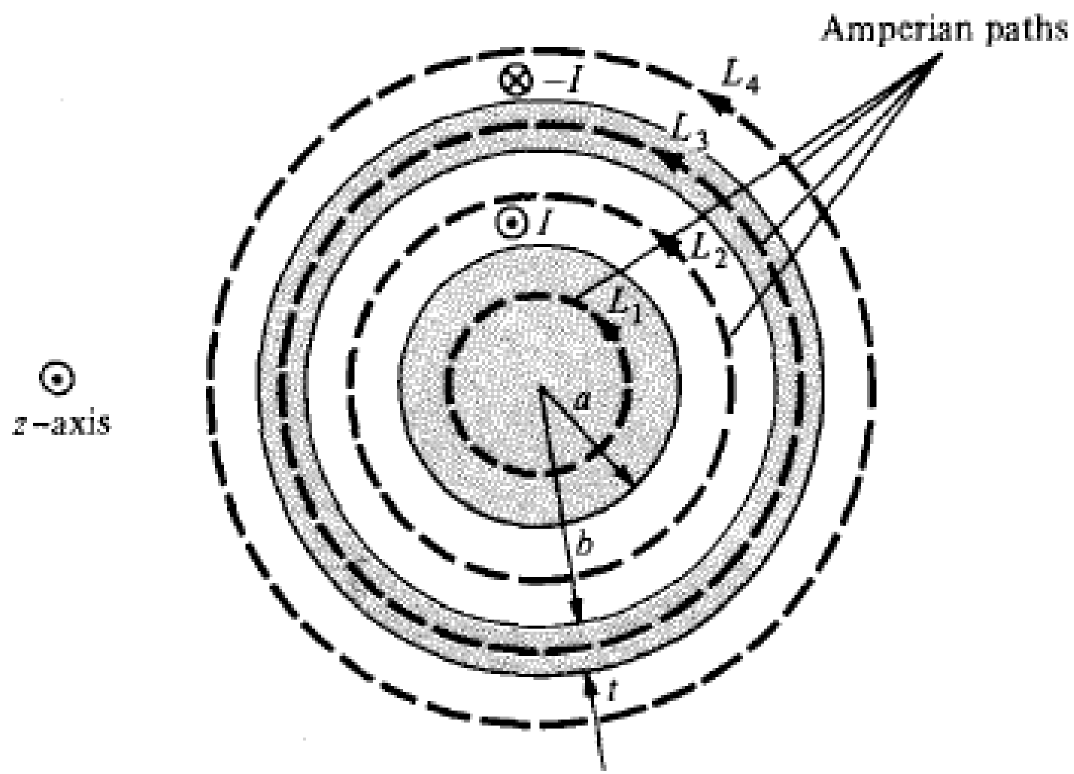
$$\vec{\mathbf{H}}_2 = \frac{-\,i\,a^2}{2\left(a^2+1\right)^{3/2}}\ \hat{a}_z \ , \quad due\,to\ symmerty\ \hat{a}_\rho - components\ vanishes$$

$$Hence \quad \vec{\mathbf{H}}_T = \vec{\mathbf{H}}_1 + \vec{\mathbf{H}}_2 = \frac{-\,i\,a^2}{\left(a^2+1\right)^{3/2}}\ \hat{a}_z$$

7-7-3: Infinite Long Coaxial Transmission Line

Consider an infinitely long transmission line consisting of two concentric cylinders having their axes along the z-axis. The cross section of the line is shown in Figure below. Where the z-axis is out of the page. The inner conductor has radius **(*a*)** and carries current **(I)** while the outer conductor has inner radius **(*b*)** and thickness **(*t*)** and carries return current **(- I)** . We want to determine H everywhere assuming that current is uniformly distributed in both conductors. Since the current distribution is symmetrical, we apply Ampere's law along the Amperian path for each of the four possible regions :

$$0 \leq \rho \leq a, a \leq \rho \leq b, b \leq \rho \leq b + t, \qquad \text{and } \rho \geq b + t.$$



(1). For region $0 \le \rho \le a$, we apply ampere's law to path (L_1), giving:

$$\oint_{L_1} \mathbf{H} \cdot d\mathbf{l} = I_{enc} = \int \mathbf{J} \cdot d\mathbf{S}$$

since the current is uniformly distributed over the cross section, hence:

$$\oint_c \vec{\mathbf{H}} \cdot d\vec{l} = \mathbf{I}_{enc} \quad , \quad \vec{\mathbf{H}} = \mathbf{H}_\phi \ \hat{a}_\phi \quad and \quad d\vec{l} = \rho \ d\phi \ \hat{a}_\phi$$

$$L.H.S. \quad \oint_c \mathbf{H}_\phi \ \hat{a}_\phi \cdot \rho \ d\phi \ \hat{a}_\phi = \mathbf{H}_\phi (\ \rho \ 2\pi) \ ------------(1)$$

$$R.H.S \quad \mathbf{I}_{enc} = \oint_s \vec{\mathbf{J}} \cdot d\vec{s} \quad , \quad \vec{\mathbf{J}} = \frac{\mathbf{I}}{\pi \ a^2} \ \hat{a}_z \quad and \quad d\vec{s} = \rho \, d\rho \, d\phi \ \hat{a}_z$$

$$\mathbf{I}_{enc} = \int_0^{2\pi} \int_0^{\rho} \frac{\mathbf{I}}{\pi \ a^2} \hat{a}_z \cdot \rho \, d\rho \, d\phi \ \hat{a}_z = \frac{\mathbf{I}}{\pi \ a^2} \ (\frac{\rho^2}{2})(2\pi) = \frac{\mathbf{I} \, \rho^2}{a^2} \ ------(2)$$

Equating the right and left hand side we get:

$$\mathbf{H}_\phi (2\pi\rho) = \mathbf{I} \frac{\rho^2}{a^2} \Rightarrow\Rightarrow \mathbf{H}_\phi = \frac{\mathbf{I}\,\rho}{2\pi a^2}$$

$$\therefore \quad \vec{\mathbf{H}} = \frac{\mathbf{I}\,\rho}{2\pi a^2} \ \hat{a}_\phi \quad for \ \ 0 \le \rho \le a$$

(2). For region $a \le \rho \le b$, we use the path (L_2) as the Amperian path:

$$\oint_c \vec{\mathbf{H}} \cdot d\vec{l} = \mathbf{I}_{enc} \quad , \quad \vec{\mathbf{H}} = \mathbf{H}_\phi \ \hat{a}_\phi \quad and \quad d\vec{l} = \rho \ d\phi \ \hat{a}_\phi$$

$$L.H.S. \quad \oint_c \mathbf{H}_\phi \ \hat{a}_\phi \cdot \rho \ d\phi \ \hat{a}_\phi = \mathbf{H}_\phi \, (\ \rho \ 2\pi \ ) \ ------------(1)$$

$$R.H.S \quad \mathbf{I}_{enc} = \oint_s \vec{\mathbf{J}} \cdot d\vec{s} \quad , \quad \vec{\mathbf{J}} = \frac{\mathbf{I}}{\pi \ a^2} \ \hat{a}_z \quad and \quad d\vec{s} = \rho \, d\rho \, d\phi \ \hat{a}_z$$

$$\mathbf{I}_{enc} = \int_0^{2\pi}\int_0^{a} \frac{\mathbf{I}}{\pi \ a^2} \hat{a}_z \cdot \rho \, d\rho \, d\phi \ \hat{a}_z = \frac{\mathbf{I}}{\pi \ a^2} \left(\frac{a^2}{2}\right)(2\pi) = \mathbf{I} \ ------(2)$$

Equating the right and left hand side we get:

$$\mathbf{H}_\phi (2\pi \rho) = \mathbf{I} \quad \Rightarrow\Rightarrow \mathbf{H}_\phi = \frac{\mathbf{I}}{2\pi \rho}$$

$$\therefore \quad \vec{\mathbf{H}} = \frac{\mathbf{I}}{2\pi \rho} \ \hat{a}_\phi \quad for \ \ a \le \rho \le b$$

(3). For region $b \le \rho \le b+t$, we use path (L_3), getting:

$$\oint_c \vec{\mathbf{H}} \cdot d\vec{l} = \mathbf{I}_{enc} \qquad , \qquad \vec{\mathbf{H}} = \mathbf{H}_\phi \, \hat{a}_\phi \qquad and \qquad d\vec{l} = \rho \, d\phi \, \hat{a}_\phi$$

$$L.H.S. \qquad \oint_c \mathbf{H}_\phi \, \hat{a}_\phi \cdot \rho \, d\phi \, \hat{a}_\phi = \mathbf{H}_\phi (\rho \, 2\pi) \text{ -----------(1)}$$

$$R.H.S \qquad \mathbf{I}_{enc} = \mathbf{I} + \oint_s \vec{\mathbf{J}} \cdot d\vec{s} = \mathbf{I} - \mathbf{I}'_{enc} \qquad \mathbf{I}'_{enc} = \oint_{s'} \vec{\mathbf{J}}' . d\vec{s}$$

$$where \quad \vec{\mathbf{J}}' = \frac{\mathbf{I}}{\pi \, [(b+t)^2 - b^2]} \hat{a}_z$$

$$and \qquad d\vec{s} = \rho \, d\rho \, d\phi \, \hat{a}_z$$

$$\mathbf{I}_{enc} = \mathbf{I} - \int_0^{2\pi} \int_b^{\rho} \frac{\mathbf{I}}{\pi \, [(b+t)^2 - b^2]} \hat{a}_z \cdot \rho \, d\rho \, d\phi \, \hat{a}_z$$

$$= \mathbf{I} - \frac{\mathbf{I}}{\pi \, [(b+t)^2 - b^2]} \left(\frac{\rho^2}{2}\right)\Big|_b^{\rho} (2\pi)$$

$$\therefore \; \mathbf{I}_{enc} = \mathbf{I} - \frac{\mathbf{I}(\rho^2 - b^2)}{(2bt + t^2)} \text{ ------(2)}$$

Equating the right and left hand side we get:

$$\mathbf{H}_\phi (2\pi\rho) = \mathbf{I} - \frac{\mathbf{I} \, (\rho^2 - b^2)}{(2bt + t^2)} \Rightarrow\Rightarrow \mathbf{H}_\phi = \frac{\mathbf{I}}{2\pi\rho} \left[1 - \frac{(\rho^2 - b^2)}{(2bt + t^2)} \right]$$

$$\therefore \; \vec{\mathbf{H}} = \frac{\mathbf{I}}{2\pi\rho} \left[1 - \frac{(\rho^2 - b^2)}{(2bt + t^2)} \right] \hat{a}_\phi \quad for \;\; b \le \rho \le c$$

(4). For region $\rho \geq (b+t)$

$$\oint_c \vec{\mathbf{H}} \cdot d\vec{l} = \mathbf{I}_{enc} \quad , \quad \vec{\mathbf{H}} = \mathbf{H}_\phi \ \hat{a}_\phi \quad and \quad d\vec{l} = \rho \ d\phi \ \hat{a}_\phi$$

$$L.H.S. \quad \oint_c \mathbf{H}_\phi \ \hat{a}_\phi \cdot \rho \ d\phi \ \hat{a}_\phi = \mathbf{H}_\phi (\rho \ 2\pi) \ ------------(1)$$

$$R.H.S \quad \mathbf{I}_{enc} = \mathbf{I} - \mathbf{I} = 0 \ ------------------------(2)$$

Hence $\vec{\mathbf{H}} = 0$ **Outside the coaxial transmission line**

Therefore, the final form of the magnetic field intensity in all region are given written generally as:

$$\mathbf{H} = \begin{cases} \dfrac{I\rho}{2\pi a^2} \mathbf{a}_\phi, & 0 \leq \rho \leq a \\ \dfrac{I}{2\pi\rho} \mathbf{a}_\phi, & a \leq \rho \leq b \\ \dfrac{I}{2\pi\rho} \left[1 - \dfrac{\rho^2 - b^2}{t^2 + 2bt} \right] \mathbf{a}_\phi, & b \leq \rho \leq b + t \\ 0, & \rho \geq b + t \end{cases}$$

The magnitude of (H) is sketched below: