7-8: Magnetic flux Density & Gauss's Law in Magnetostatic Fields

The magnetic flux density (B) is similar to the electric flux density (D). As ($\mathbf{D} = \varepsilon_\circ \mathbf{E}$) in free space, the magnetic flux density is related to the magnetic field intensity **(H)** according to:

$$\mathbf{B} = \mu_\circ \mathbf{H}$$

where : $\mu_\circ = Free\ space\ permiability = 4\pi \times 10^{-7}\ (H/m)$

(H): is the magnetic field intensity measure in unit of (A/m)

(B): is the magnetic flux density measure in unit of (Weber/ m^2)=Tesla= T

$$1\ Weber = 10^8\ Maxwell \qquad and \qquad 1\ m^2 = 10^4\ cm^2$$

$$1\ \frac{Weber}{m^2} = 1\ Tesla = \frac{10^8\ Maxwell}{10^4\ cm^2} = 10^4\ Gauss$$

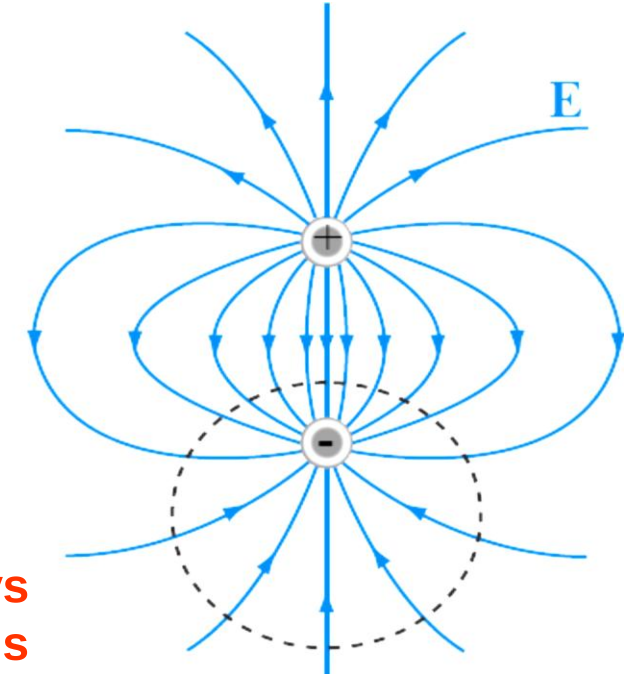
$$\therefore\ 1\ Tesla = 10^4\ Gauss$$

In a region contain a magnetic field; we can determine the magnetic flux passing through any closed surface surrounding that region by the following equation:

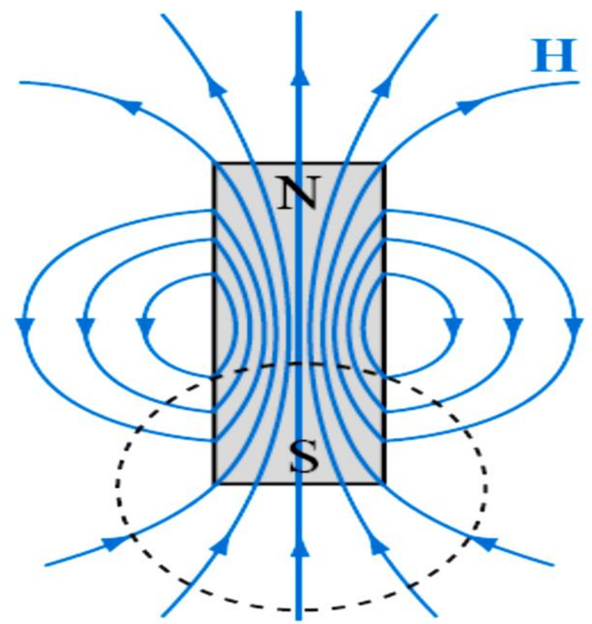
$$\psi_m = \Phi = \int \vec{\mathbf{B}} \cdot \overrightarrow{ds}\ and \quad \Phi\ is\ measure\ in\ unit\ of\ (Weber)\ or\ (Maxwell)$$

In electrostatic field, the electric flux passing through a closed surface is the same as the charge enclosed. Thus it is possible to have an isolated electric charge according to:

$$\psi_e = \oint_s \vec{\mathbf{D}} \cdot \overrightarrow{ds} = Q_{enc}$$



In magnetostatic field, the magnetic flux line always closes upon themselves. This is due to the fact that it is impossible to have magnetic monopole.



According to the above figures, the total flux passing through any closed surface in magnetostatic field must be zero and this written mathematically as:

$$\oint_s \vec{\mathbf{B}} \cdot \overrightarrow{ds} = 0$$

For Magnetostatic Field and according to divergence theorem, we have:

$$\oint_S \vec{\mathbf{A}} \cdot \overrightarrow{ds} = \int_v (\vec{\nabla} \cdot \vec{\mathbf{A}})\, dv$$

Therefore,

$$\oint_S \vec{\mathbf{B}} \cdot \overrightarrow{ds} = \int_v (\vec{\nabla} \cdot \vec{\mathbf{B}})\, dv = 0$$

$$Hence: \quad \vec{\nabla} \cdot \vec{\mathbf{B}} = 0$$

This equation ($\vec{\nabla} \cdot \vec{\mathbf{B}} = 0$) is the fourth Maxwell's equation and is called the law of conservation of magnetic flux or point form of Gauss's law in magnetostatic field.

Physically, this equation means that:

(1). [The existence of magnetic monopole is impossible]

(2). [Magnetic field has no source and sink]

Therefore, the final form of Maxwell's equations in electrostatic and magnetostatic field in both differential and integral form can be written as:

Differential form (Point form)	Integral form	Remarks
$\vec{\nabla} \cdot \vec{\mathbf{D}} = \rho_v$	$\oint_s \vec{\mathbf{D}} \cdot \overrightarrow{ds} = Q_{enc} = \int_v \rho_v dv$	Gauss's Law in electrostatic field
$\vec{\nabla} \cdot \vec{\mathbf{B}} = 0$	$\oint_s \vec{\mathbf{B}} \cdot \overrightarrow{ds} = 0$	Gauss's law in magnetostatic field, non-existence of magnetic monopole
$\vec{\nabla} \times \vec{\mathbf{E}} = 0$	$\oint_c \vec{\mathbf{E}} \cdot \overrightarrow{dl} = 0$	Conservation of electrostatic field intensity
$\vec{\nabla} \times \vec{\mathbf{H}} = \vec{\mathbf{J}}$	$\oint_c \vec{\mathbf{H}} \cdot \overrightarrow{dl} = \mathbf{I}_{enc} = \int_s \vec{\mathbf{J}} \cdot \overrightarrow{ds}$	Ampere's law

7-9: Magnetic Scalar and Vector Potentials

We recall that some electrostatic field problems were simplified by relating the electric field potential **(V)** to the electric field intensity **(E)** through the relation:

$$\vec{\mathbf{E}} = -\vec{\nabla}\mathbf{V}$$

Similarly, we can define a potential associated with magnetostatic field. In fact, the magnetic potential could be scalar (V_m) or vector ($\vec{\mathbf{A}}$)

The concept of ($\vec{\mathbf{E}} = -\vec{\nabla}\mathbf{V}$) have been found from the identity ($\vec{\nabla}\times\vec{\nabla}f = 0$) where **(f)** is any scalar function and ($\vec{\nabla}\times\vec{\mathbf{E}} = 0$) in electrostatic field. From these two equations we note that there must exist a scalar function such as **(f)** whose gradient can be make equal to **(E)**. If we make **(** $f = -\mathbf{V}$ **)** we obtain the relation:

$$\vec{\mathbf{E}} = -\vec{\nabla}\mathbf{V}$$

In parallel manner, we have an identity that the divergence of curl of nay vector field is equal to zero as given below:

$$\vec{\nabla}\cdot\vec{\nabla}\times\vec{\mathbf{A}} = 0 \quad ----(1)$$

From the Maxwell's equation in magnetostatic field we have:

$$\vec{\nabla}\cdot\vec{\mathbf{B}} = 0 \quad ------(2-a)$$

$$\vec{\nabla}\times\vec{\mathbf{H}} = \vec{\mathbf{J}} \quad ------(2-b)$$

Just as ($\vec{\mathbf{E}} = -\vec{\nabla}\mathbf{V}$), we define the magnetic scalar potential **(V_m)**, in unit of **(Ampere)** as related to the magnetic field intensity **(H)** according to:

$$\vec{\mathbf{H}} = -\vec{\nabla}\mathbf{V}_m \qquad if \quad \mathbf{J} = 0 \quad ------------(3)$$

$$\mathbf{V}_m = is\ the\ magnetic\ Scalar\ potential\ in\ (Ampere)$$

Since, if $\mathbf{J} \neq 0 \qquad the\ \ identity\ \ \vec{\nabla}\times\vec{\mathbf{H}} = \vec{\nabla}\times\vec{\nabla}\mathbf{V}_m \neq 0$

Therefore, the magnetic scalar potential (V_m) is defined only in a region where (J = 0). We should also note that (V_m) satisfies Laplace's equation:

$$\nabla^2\mathbf{V}_m = 0 \qquad for \qquad \mathbf{J} = 0 \quad -------(4)$$

Assuming that the magnetic flux density is given by:

$$\vec{\mathbf{B}} = \vec{\nabla}\times\vec{\mathbf{A}} \quad ------------(5) \qquad where \quad \vec{\nabla}\cdot\vec{\mathbf{A}} = 0$$

$$\vec{\mathbf{A}} = is\ called\ Mganteic\ Vector\ potential\ in\ (Weber/m)$$

$$\therefore \quad \vec{\nabla}\cdot\vec{\mathbf{B}} = \vec{\nabla}\cdot\vec{\nabla}\times\vec{\mathbf{A}} = 0$$

$$\vec{\nabla} \times \vec{\mathbf{H}} = \vec{\mathbf{J}} \qquad and \qquad \vec{\mathbf{B}} = \mu_\circ \vec{\mathbf{H}}$$

$$hence: \qquad \vec{\nabla} \times \vec{\mathbf{B}} = \mu_\circ \vec{\mathbf{J}} ------------(6)$$

$$\vec{\nabla} \times \vec{\nabla} \times \vec{\mathbf{A}} = \vec{\nabla}(\vec{\nabla} \cdot \vec{\mathbf{A}}) - \vec{\nabla}^2 \vec{\mathbf{A}} = \mu_\circ \mathbf{J} \qquad but \quad \vec{\nabla} \cdot \vec{\mathbf{A}} = 0$$

Hence,

$$\vec{\nabla}^2 \vec{\mathbf{A}} = -\mu_\circ \mathbf{J} \; ------------(7)$$

$$\nabla^2 \mathbf{V} = -\frac{\rho_v}{\varepsilon_\circ}$$

Equation (7) is called the vector Poisson's equation in magnetostatic field. In the case of no current (J= 0)in the region, eq.(7) reduces to Laplae's equation.

$$\mathbf{V} = \frac{1}{4\pi\varepsilon_\circ}\int_l \frac{\rho_l \, dl}{\mathbf{R}} \quad for \; line \; charge$$

$$\mathbf{V} = \frac{1}{4\pi\varepsilon_\circ}\int_s \frac{\rho_s \, ds}{\mathbf{R}} \quad for \; surface \; charge$$

$$\mathbf{V} = \frac{1}{4\pi\varepsilon_\circ}\int_v \frac{\rho_v \, dv}{\mathbf{R}} \quad for \; volume \; charge$$

$$\vec{\mathbf{A}} = \frac{\mu_\circ}{4\pi}\int_l \frac{\mathbf{I} \, d\vec{l}}{\mathbf{R}} \quad for \; filamentary \; current \; density$$

$$\vec{\mathbf{A}} = \frac{\mu_\circ}{4\pi}\int_s \frac{\vec{\mathbf{K}} \, ds}{\mathbf{R}} \quad for \; surface \; current \; density$$

$$\vec{\mathbf{A}} = \frac{\mu_\circ}{4\pi}\int_v \frac{\vec{\mathbf{J}} \, dv}{\mathbf{R}} \quad for \; volume \; current \; density$$

Therefore, the magnetic flux (Φ) passing through any surface surrounding the region contain magnetic field can be calculated through one of the following equation:

$$\Phi = \oint_s \vec{\mathbf{B}} \cdot \overrightarrow{ds} = \oint_s (\vec{\nabla} \times \vec{\mathbf{A}}) \cdot \overrightarrow{ds} \quad ---------(8)$$

And according to the divergence, we have:

$$\oint_l \vec{\mathbf{A}} \cdot \overrightarrow{dl} = \oint_s (\vec{\nabla} \times \vec{\mathbf{A}}) \cdot \overrightarrow{ds}$$

Thus, eq.(8) reduces to:

$$\Phi = \oint_l \vec{\mathbf{A}} \cdot \overrightarrow{dl} \quad -----(9)$$

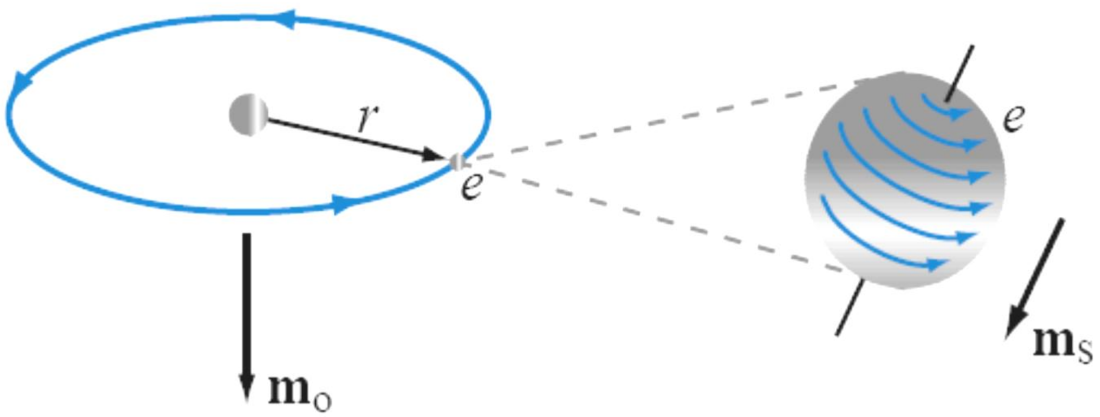
7-10: Magnetization and Classification of Magnetic Materials

The magnetic moment (m) of a current loop of area (A) has a magnitude of (m = I A), and the direction of (m) is normal to the plane of the loop in accordance with the right hand rule. Magnetization in a material substance is associated with atomic current loops generated by three principal mechanisms:

(1). Orbital motions of the electrons around the nucleus and similar motions of the protons around each other in the nucleus,

(2). Spin motion of electrons around itself

(3). Nuclear Spin magnetic moment



(a) Orbiting electron (b) Spinning electron

The circular motion of an electron produce a current loop (I) around an area of ($A = \pi r^2$), and is given by:

$$\mathbf{I} = -\frac{e}{T} = -e\ f \qquad and \qquad T = \frac{2\pi r}{u}$$

where (u) is the speed of electron

(T) is the time period

(f) is the oscilation frequency

$$\mathbf{I} = -\frac{e\,u}{2\pi r} -------------(1)$$

The magnitude of the associated **orbital magnetic moment** ($\mathbf{m}_l$) is given by:

$$\vec{\mathbf{m}}_l = \mathbf{I}\ A = -\left(\frac{e\,u}{2\pi r}\right)\left(\pi r^2\right) = -\frac{e\,u\,m_e\,r}{2m_e}$$

and $\vec{\mathbf{L}} = m_e\,u\,r$ *orbital angular momentum*

$$\vec{\mathbf{m}}_l = -\frac{e}{2m_e}\vec{\mathbf{L}} -------------(2)$$

and $\vec{\mathbf{L}} = n\hbar$ $-------------(3)$

The smallest nonzero magnitude of the orbital magnetic moment of an electron is:

$$\mathbf{m}_l = -\frac{e\hbar}{2m_e} -------------(4)$$

In addition to the magnetic moment produced by its orbital motion, an electron generates a spin magnetic moment **(m_s)** due to its spinning motion about its own axis as shown in the above figure. The magnitude of **(m_s)** produced by quantum theory is:

$$\vec{\mathbf{m}}_s = -\frac{e}{m_e}\vec{\mathbf{S}} ------(5)$$

$$\mathbf{m}_s = \pm\frac{e\hbar}{2m_e} -------(6)$$

Despite the fact that all substances contain electrons and the electrons exhibits magnetic dipole moments, most substances are effectively nonmagnetic. This is because, in the absence of an external magnetic field, the atoms of most materials are oriented randomly, as a result of which the net magnetic moment generated by their electrons is either zero or very small.

The magnetic behavior of a material is governed by the interaction of the magnetic dipole moments of its atoms with an external magnetic field. This behavior, which depends on the crystalline structure of the material, is used as a basis for classifying materials as:

(1). Diamagnetic materials
(2). Paramagnetic materials
(3). Ferro-magnetic materials
(4). Anti-ferromagnetic materials
(5). Ferri-magnetic materials
(6). Supermagnetic materials

Diamagnetic materials: The diamagnetic materials are substances which have atoms or ions with complete shells or even numbers of free electrons, in which the orbital and spin magnetic moments cancel each other making net permanent magnetic moment of each atom equal to zero. Hence, their diamagnetic behavior is due to the fact that the external magnetic field acts to distort the orbital motion.

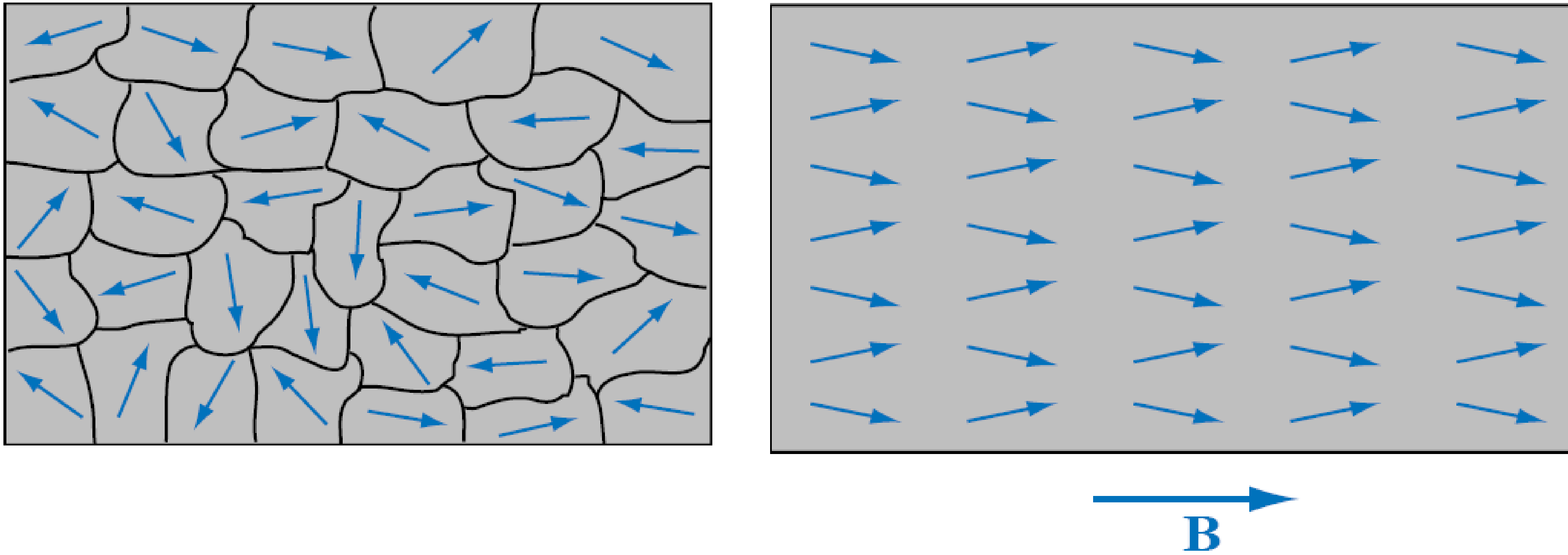
Paramagnetic materials : The paramagnetic materials are substances which have atoms or ions with incomplete shells or have odd numbers of free electrons, and their paramagnetic behavior is due to the combination of the orbital and spin motions of the free electrons.

Ferromagnetic materials: The ferromagnetism is a phenomenon of spontaneous magnetization, which involves the alignment of an appreciable fraction of the molecular magnetic moments in some favorable direction in the crystal, which have some domain of magnetic moments distributed randomly. Or in the ferromagnetic materials the adjacent atoms line up their magnetic dipole moments in parallel fashion in the lattice.

Anti-ferromagnetic materials : The materials in which the magnetic dipole moments of adjacent atoms line up in anti-parallel fashion are called anti-ferromagnetic materials. The net magnetic moment in such materials is zero. Such materials when placed near a a strong magnets gets neither attracted nor repelled.

ferrimagnetic materials : The materials in which the magnetic dipole moments of adjacent atoms line up in anti-parallel fashion, but the net magnetic moment is non-zero are called ferrimagnetic materials. Such materials when placed near a strong magnets gets affected by an order lower than ferromagnetic materials.

Super-magnetic materials : The ferromagnetic materials when suspended in the dielectric matrixes is become a supermagnetic materials in which even though each particles of it contain large magnetic domain but can not penetrate adjacent particles. The common example of such materials are magnetic tapes used for audio, video and data recording.



	Diamagnetism	Paramagnetism	Ferromagnetism
Permanent magnetic dipole moment	No	Yes, but weak	Yes, and strong
Primary magnetization mechanism	Electron orbital magnetic moment	Electron spin magnetic moment	Magnetized domains
Direction of induced magnetic field (relative to external field)	Opposite	Same	Hysteresis [see Fig. 5-22]
Common substances	Bismuth, copper, diamond, gold, lead, mercury, silver, silicon	Aluminum, calcium, chromium, magnesium, niobium, platinum, tungsten	Iron, nickel, cobalt
Typical value of χ_m **Typical value of μ_r**	$\approx -10^{-5}$ ≈ 1	$\approx 10^{-5}$ ≈ 1	$\lvert\chi_m\rvert \gg 1$ and hysteretic $\lvert\mu_r\rvert \gg 1$ and hysteretic

The magnetization vector (M), of a material is defined as the vector sum of the magnetic dipole moments of the atoms contained in a unit volume of the material and measure in a unit of (A/m).

$$\vec{\mathbf{M}} = \frac{\sum_{i=1}^{n} \mathbf{m}_i}{\Delta v} \qquad in\ unit\ \ (A/m) \ ---------(7)$$

$$\mathbf{M} = \chi_m \mathbf{H} \ -----------(8) \qquad where, \quad (\chi_m)\ is\ \ called\ \ magneic\ \ susssceptiblitiy$$

Therefore, the magnetic flux density inside a material is the sum of the magnetic field in free space and that produced as magnetization as given below:

$$\mathbf{B} = \mu_\circ \mathbf{H} + \mu_\circ \mathbf{M} \qquad and \qquad \mathbf{M} = \chi_m \mathbf{H}$$

$$\mathbf{B} = \mu_\circ (\mathbf{H} + \chi_m \mathbf{H})$$

$$\mathbf{B} = \mu_\circ (1 + \chi_m)\ \mathbf{H} \qquad = \mu\ \mathbf{H} \ ----(9)$$

$$where, \quad \mu = \mu_\circ\ \mu_r = \mu_\circ (1 + \chi_m)\ -(10)$$

$$and \qquad \mu_r = (1 + \chi_m) --------(11)$$

$$\mu_r : \mathrm{Re}lative\ \ permiability$$

$$\mu : Permiability\ of\ \ the\ \ material\ \ Medium$$

$$\mu_\circ = 4\pi \times 10^{-7}\ \ (H/m)$$