Chapter Seven

Stationary Waves and Interference

Reflection, refraction and diffraction

7.1: **Stationary Waves:**

- **When two simple harmonic waves of the same amplitude, frequency and time period travel in opposite directions in a straight line, the resultant wave obtained is called a *stationary or a standing wave*.**

At an instant of time *t=0*, the waves are as shown in figure (1). The resultant displacement curve is a straight line. All the particles of the medium are at their mean positions.

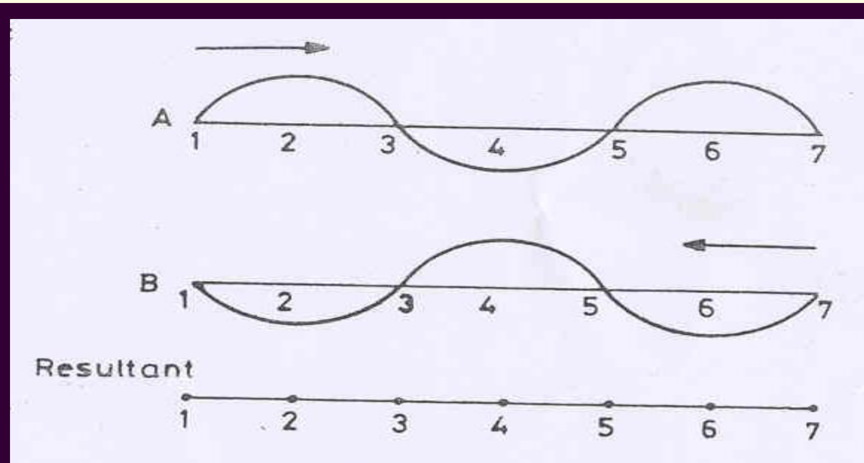


Fig. 6.1.

At time *t=T/4*, the wave *A* will advance through a distance *λ/4* towards right and the wave *B* will advance through a distance *λ/4* towards left. The resultant displacement pattern is shown in figure (2).

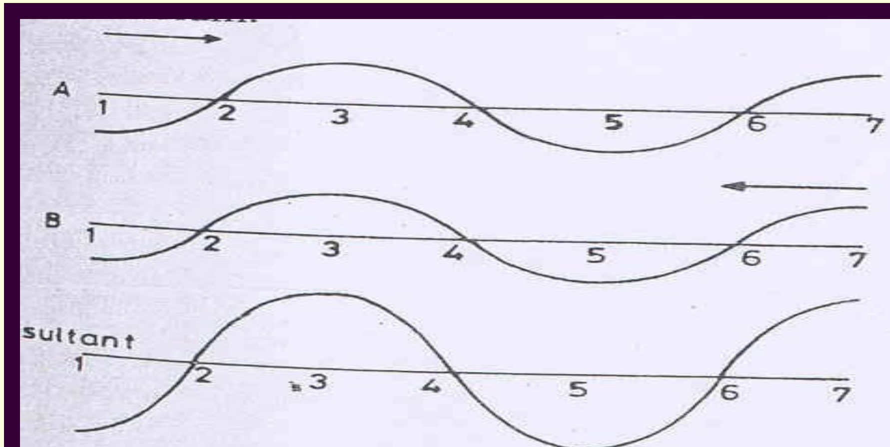


Fig. 6.2.

The particles 1, 3, 5 and 7 are at their extreme positions and particles 2, 4 and 6 are at their mean position.

At time *t=T/2*, the wave *A* will advance through a distance *λ/2* towards right and the wave *B* will advance through a distance *λ/2* towards left (with reference to zero time).The resultant displacement pattern is shown in figure (3).

All the particles of the medium are at their mean positions.

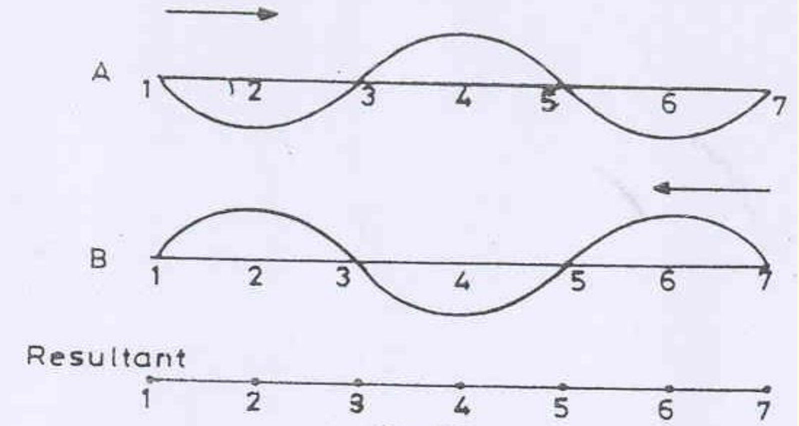


Fig. 6.3.

At time *t=3T/4*, the wave *A* will advance through a distance *3λ/4* towards right and the wave *B* will advance through a distance *3λ/4* towards left (with reference to zero time).The resultant displacement pattern is shown in figure (4).

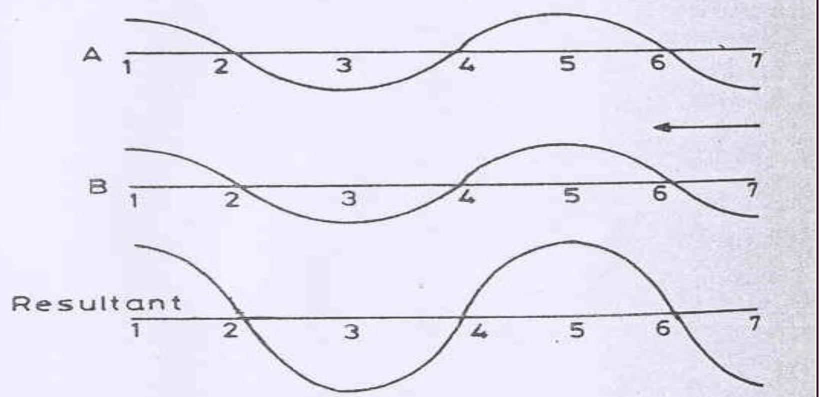


Fig. 6.4.

The particles 1, 3, 5 and 7 are at their extreme positions and 2, 4, 6 are at their mean positions.

At time $t=T$, the wave A will advance through a distance λ towards right and the wave B will advance through a distance λ towards left (with reference to zero time). The resultant displacement pattern is shown in figure (5).

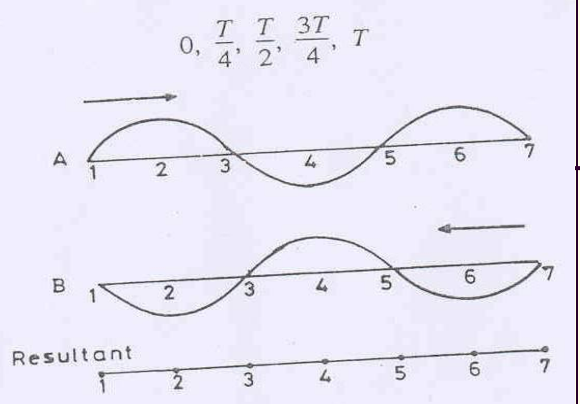


Fig. 6.5.

From the patterns discussed above, it is clear that the particles of the medium such as 2, 4, 6 etc. always remain at their mean positions. The particles such as 1, 3, 5, 7 etc. continue to vibrate simple harmonically about their mean positions with double the amplitude of each wave. It appears as through the wave pattern is stationary in space. The resultant displacement patterns at in travels of time $0, \frac{T}{4}, \frac{T}{2}, \frac{3T}{4}$ $and\ T$ are shown in figure (6).

The positions of the particles 1, 3, 5, 7 etc. which vibrate simple harmonically with maximum amplitude (twice the amplitude of each wave) are called antinodes. At the antinodes, the strain is a minimum. The distance between any two consecutive nodes or antinodes is equal to $\lambda/2$. Between a node and antinodes, the amplitude gradually increases from zero to maximum.

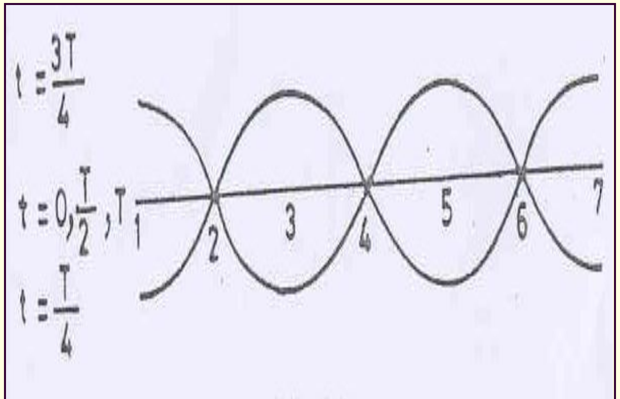


Fig. 6.6.

The positions of the particles 2, 4, 6 etc. which always remain at their mean positions are called nodes. Node is a position of zero displacement and maximum strain.

7.2: Properties of Stationary Longitudinal Waves:

- **The important properties of these waves are;**

(1) In these waves, nodes and antinodes are formed alternately. Nodes are the positions where the particles are at their mean positions having maximum strain. Antinodes are the positions where the particles vibrate with maximum amplitude having minimum strain .

(2) All the particles except at the nodes vibrate simple harmonically with the time period equal to that of each component wave.

(3) The amplitude of vibration gradually increases from zero to maximum from node to antinodes.

(4) The medium is split into segments and all the particles of the same segment vibrate in phase. The particles in one segment have a phase difference of λ with the particles in the neighboring segment.

(5) Condensations and rarefactions do not travel forward as in progressive waves but they appear and disappear alternately at the same place.

(6) The distance between two adjacent nodes isλ/2 and also the distance between two adjacent antinodes is λ/2. The distance between a node and the adjacent antinodes is λ/4. Between two nodes there is an antinodes and vice versa.

(7) The general appearance of the wave can be represented by a sine curve but it reduces to straight line twice in each time period.

(8) Velocity and acceleration of all the particles separated by a distance λ are the same at a given instant.

(9) In the same segment, at the same instant, all the particles will be in phase and their velocities and accelerations will be maximum or minimum at the same instant.

7.3: Analytical Treatment:

- Stationary waves are formed in an open end organ pipe or a closed end organ pipe. Stationary waves are also formed with a stretched string fixed one end and free at the other end or fixed at the other end.

(a) Open end organ pipe or string free at the end:

$$y_1 = a\sin\frac{2\pi}{\lambda}(\upsilon t - x) \ \ (Incident\ wave) - - - (1)$$

$$y_2 = a\sin\frac{2\pi}{\lambda}(\upsilon t + x) \ \ (\text{Re}\ flected\ wave) - - - (2)$$

$$y = y_1 + y_2$$

$$y = a\sin\frac{2\pi}{\lambda}(\upsilon t - x) + a\sin\frac{2\pi}{\lambda}(\upsilon t + x) - - - (3)$$

$$= 2\ a\ \cos\frac{2\pi x}{\lambda}\sin\frac{2\pi\upsilon t}{\lambda} - - - (4)$$

$$A = 2\ a\ \cos\frac{2\pi x}{\lambda} - - - (5)$$

$$\frac{dy}{dt} = \frac{-4\pi\ a\upsilon}{\lambda}\cos\frac{2\pi x}{\lambda}\cos\frac{2\pi\upsilon t}{\lambda} - - - (6)$$

$$\frac{d^2 y}{dt^2} = \frac{8\pi^2 a\upsilon^2}{\lambda^2}\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi\upsilon t}{\lambda} - - - (7)$$

$$\frac{dy}{dx} = \frac{-4\pi a}{\lambda}\sin\frac{2\pi x}{\lambda}\sin\frac{2\pi\upsilon t}{\lambda} - - - (8)$$

Changes with respect to position:

- **(i) Consider the positions, where** $\sin\frac{2\pi x}{\lambda}=0 \quad and \quad \cos\frac{2\pi x}{\lambda}=\pm 1$

$$y = 2a\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi \upsilon t}{\lambda} --- (4)$$

$$A = 2a\cos\frac{2\pi x}{\lambda} --- (5)$$

$$\frac{dy}{dt} = \frac{-4\pi a\upsilon}{\lambda}\cos\frac{2\pi x}{\lambda}\cos\frac{2\pi \upsilon t}{\lambda} --- (6)$$

$$\frac{d^2y}{dt^2} = \frac{8\pi^2 a\upsilon^2}{\lambda^2}\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi \upsilon t}{\lambda} --- (7)$$

$$\frac{dy}{dx} = \frac{-4\pi a}{\lambda}\sin\frac{2\pi x}{\lambda}\sin\frac{2\pi \upsilon t}{\lambda} --- (8)$$

$$y = \pm 2a\sin\frac{2\pi \upsilon t}{\lambda} --- (9)$$

$$A = \pm 2a --- (10)$$

$$\frac{dy}{dt} = \frac{\pm 4\pi a\upsilon}{\lambda}\cos\frac{2\pi \upsilon t}{\lambda} --- (11)$$

$$\frac{d^2y}{dt^2} = \frac{\mp 8\pi^2 a\upsilon^2}{\lambda^2}\sin\frac{2\pi \upsilon t}{\lambda} --- (12)$$

$$\frac{dy}{dx} = 0 --- (13)$$

$$\sin\frac{2\pi x}{\lambda} = 0$$

But sin (mπ) =0 Where m=0, 1, 2, 3… etc.

$$\frac{2\pi x}{\lambda} = m\pi \quad or \quad x = \frac{m\lambda}{2}$$

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2} ...etc.$$

Thus, the antinodes are equidistant and separated by λ/2. At x=0, the position of interface is an antinodes. Therefore, the position of the open end of the open end organ pipe or free end of a stretched string corresponds to antinodes.

Changes with respect to position:

- **(ii) Consider the positions, where** $\sin\frac{2\pi x}{\lambda}=\pm 1 \quad and \quad \cos\frac{2\pi x}{\lambda}=0$

$$y=2a\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi \upsilon t}{\lambda}---(4)$$

$$A=2a\cos\frac{2\pi x}{\lambda}---(5)$$

$$\frac{dy}{dt}=\frac{-4\pi a\upsilon}{\lambda}\cos\frac{2\pi x}{\lambda}\cos\frac{2\pi \upsilon t}{\lambda}---(6)$$

$$\frac{d^2y}{dt^2}=\frac{8\pi^2 a\upsilon^2}{\lambda^2}\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi \upsilon t}{\lambda}---(7)$$

$$\frac{dy}{dx}=\frac{-4\pi a}{\lambda}\sin\frac{2\pi x}{\lambda}\sin\frac{2\pi \upsilon t}{\lambda}---(8)$$

Displacement; y =0 ------ (14)

Amplitude; A =0 ------ (15)

Velocity; dy/dt =0 ----- (16)

Acceleration; d^2y/dt^2 =0 ---- (17)

$$\frac{dy}{dx}=\frac{\mp 4\pi a}{\lambda}\sin\frac{2\pi \upsilon t}{\lambda}---(18)$$

$$\cos\frac{2\pi x}{\lambda}=0$$

But $\cos(2m+1)\frac{\pi}{2}=0$ Where m=0, 1, 2, 3… etc.

$$(2m+1)\frac{\pi}{2}=\frac{2\pi x}{\lambda} \quad or \quad x=\frac{(2m+1)\lambda}{4}$$

$$\boxed{x=\frac{\lambda}{4},\frac{3\lambda}{4},\frac{5\lambda}{4}......etc.}$$

Thus, the antinodes are also equidistant and separated by a distance of λ/2. It is clear between consecutive antinodes there is a node and vice versa. Distance between a node and an antinodes is λ/4.

Changes with respect to time :

- (i) Consider the instant to time, when $\sin\frac{2\pi\upsilon t}{\lambda}=0 \quad and \quad \cos\frac{2\pi\upsilon t}{\lambda}=\pm 1$

$$y=2\,a\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi\upsilon t}{\lambda}---(4)$$

$$A=2\,a\cos\frac{2\pi x}{\lambda}---(5)$$

$$\frac{dy}{dt}=\frac{-4\pi\,a\upsilon}{\lambda}\cos\frac{2\pi x}{\lambda}\cos\frac{2\pi\upsilon t}{\lambda}---(6)$$

$$\frac{d^2y}{dt^2}=\frac{8\pi^2a\upsilon^2}{\lambda^2}\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi\upsilon t}{\lambda}---(7)$$

$$\frac{dy}{dx}=\frac{-4\pi a}{\lambda}\sin\frac{2\pi x}{\lambda}\sin\frac{2\pi\upsilon t}{\lambda}---(8)$$

$$y=0$$

$$A=2\,a\cos\frac{2\pi x}{\lambda}$$

$$\frac{dy}{dt}=\frac{\pm 4\pi\,a\upsilon}{\lambda}\cos\frac{2\pi x}{\lambda}$$

$$\frac{d^2y}{dt^2}=0$$

$$\frac{dy}{dx}=0$$

$$\sin\frac{2\pi\upsilon t}{\lambda}=0$$

But sin (mπ) =0 Where m=0, 1, 2, 3… etc.

$$\frac{2\pi\upsilon t}{\lambda}=m\pi \quad or \quad t=\frac{m\lambda}{2\upsilon}$$

$$\frac{\upsilon}{\lambda}=n=\frac{1}{T}$$

$$t=\frac{mT}{2} \qquad or \quad t=0,\frac{T}{2},T,\frac{3T}{2}...etc.$$

If follows that twice in each time period the particles pass through their mean positions.

Changes with respect to time:

- **(ii) Consider the instant to time, when** $\sin\frac{2\pi\upsilon t}{\lambda}=\pm 1 \quad and \quad \cos\frac{2\pi\upsilon t}{\lambda}=0$

$$y=2a\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi\upsilon t}{\lambda}---(4)$$

$$A=2a\cos\frac{2\pi x}{\lambda}---(5)$$

$$\frac{dy}{dt}=\frac{-4\pi a\upsilon}{\lambda}\cos\frac{2\pi x}{\lambda}\cos\frac{2\pi\upsilon t}{\lambda}---(6)$$

$$\frac{d^2y}{dt^2}=\frac{8\pi^2 a\upsilon^2}{\lambda^2}\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi\upsilon t}{\lambda}---(7)$$

$$\frac{dy}{dx}=\frac{-4\pi a}{\lambda}\sin\frac{2\pi x}{\lambda}\sin\frac{2\pi\upsilon t}{\lambda}---(8)$$

$$y=\pm 2a\cos\frac{2\pi x}{\lambda}$$

$$A=2a\cos\frac{2\pi x}{\lambda}$$

$$\frac{dy}{dt}=0$$

$$\frac{d^2y}{dt^2}=\frac{\mp 8\pi^2 a\upsilon^2}{\lambda^2}\cos\frac{2\pi x}{\lambda}$$

$$\frac{dy}{dx}=\frac{\mp 4\pi a}{\lambda}\sin\frac{2\pi x}{\lambda}$$

$$\sin\frac{2\pi\upsilon t}{\lambda}=\pm 1 \quad and \quad \cos\frac{2\pi\upsilon t}{\lambda}=0$$

But $\sin(2m+1)\frac{\pi}{2}=\pm 1$ Where m=0, 1, 2, 3… etc.

$$\frac{2\pi\upsilon t}{\lambda}=(2m+1)\frac{\pi}{2} \quad or \quad t=\frac{(2m+1)\lambda}{4\upsilon}$$

$$\frac{\upsilon}{\lambda}=n=\frac{1}{T}$$

$$t=\frac{(2m+1)T}{2} \quad or \quad t=0,\frac{T}{4},\frac{3T}{4},\frac{5T}{4}...etc.$$

At these instants, the displacement, acceleration and strain are a maximum at all positions and the velocity of the particles is zero. At these instants, each particle is at its extreme position and the pattern is stationary at that instant. This instant is called the stationary instant.

(b) Closed end organ pipe or string fixed at the other end:

$$y_1 = a\sin\frac{2\pi}{\lambda}(\upsilon t - x) \quad (Incident\ wave) ---(1)$$

$$y_2 = -a\sin\frac{2\pi}{\lambda}(\upsilon t + x) \quad (\mathrm{Re}\ flected\ wave) ---(2)$$

$$y = y_1 + y_2$$

$$y = -2\ a\ \sin\frac{2\pi x}{\lambda}\cos\frac{2\pi\upsilon t}{\lambda} ---(4)$$

$$A = 2\ a\ \cos\frac{2\pi x}{\lambda} ---(5)$$

$$\frac{dy}{dt} = \frac{4\pi\ a\upsilon}{\lambda}\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi\upsilon t}{\lambda} ---(6)$$

$$\frac{d^2y}{dt^2} = \frac{8\pi^2 a\upsilon^2}{\lambda^2}\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi\upsilon t}{\lambda} ---(7)$$

$$\frac{dy}{dx} = \frac{-4\pi a}{\lambda}\cos\frac{2\pi x}{\lambda}\cos\frac{2\pi\upsilon t}{\lambda} ---(8)$$

Changes with respect to position:

(i) Consider the positions, where

$$\sin\frac{2\pi x}{\lambda}=0 \quad and \quad \cos\frac{2\pi x}{\lambda}=\pm 1$$

$$y=-2a\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi\upsilon t}{\lambda}---(4)$$

$$A=2a\cos\frac{2\pi x}{\lambda}---(5)$$

$$\frac{dy}{dt}=\frac{4\pi a\upsilon}{\lambda}\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi\upsilon t}{\lambda}---(6)$$

$$\frac{d^2y}{dt^2}=\frac{8\pi^2 a\upsilon^2}{\lambda^2}\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi\upsilon t}{\lambda}---(7)$$

$$\frac{dy}{dx}=\frac{-4\pi a}{\lambda}\cos\frac{2\pi x}{\lambda}\cos\frac{2\pi\upsilon t}{\lambda}---(8)$$

Displacement; y =0 ------ (9)

Amplitude; A =0 ------ (10)

Velocity; dy/dt =0 ----- (11)

Acceleration; d2y/dt2 =0 ---- (12)

$$\frac{dy}{dx}=\frac{\mp 4\pi a}{\lambda}\cos\frac{2\pi\upsilon t}{\lambda}---(13)$$

These positions correspond to nodes. $\sin\frac{2\pi x}{\lambda}=0$ **But sin (mπ) =0 Where m=0, 1, 2, 3… etc.**

$$\frac{2\pi x}{\lambda}=m\pi \quad or \quad x=\frac{m\lambda}{2}$$

$$x=0,\frac{\lambda}{2},\lambda,\frac{3\lambda}{2}...etc.$$

Thus, the nodes are equidistant and separated by λ/2. At x=0, the position of interface is a nodes. Therefore, the position of the closed end organ pipe or fixed end of a stretched string corresponds to a node.

Changes with respect to position:
(ii) Consider the positions, where

$$\sin\frac{2\pi x}{\lambda}=\pm 1 \quad and \quad \cos\frac{2\pi x}{\lambda}=0$$

$$y=-2a\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi \upsilon t}{\lambda}---(4)$$

$$y=\pm 2a\cos\frac{2\pi \upsilon t}{\lambda}---(14)$$

$$A=2a\cos\frac{2\pi x}{\lambda}---(5)$$

$$A=\pm 2a---(15)$$

$$\frac{dy}{dt}=\frac{4\pi a\upsilon}{\lambda}\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi \upsilon t}{\lambda}---(6)$$

$$\frac{dy}{dt}=\frac{\pm 4\pi a\upsilon}{\lambda}\sin\frac{2\pi \upsilon t}{\lambda}---(16)$$

$$\frac{d^2y}{dt^2}=\frac{8\pi^2 a\upsilon^2}{\lambda^2}\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi \upsilon t}{\lambda}---(7)$$

$$\frac{d^2y}{dt^2}=\frac{\mp 8\pi^2 a\upsilon^2}{\lambda^2}\cos\frac{2\pi \upsilon t}{\lambda}---(17)$$

$$\frac{dy}{dx}=\frac{-4\pi a}{\lambda}\cos\frac{2\pi x}{\lambda}\cos\frac{2\pi \upsilon t}{\lambda}---(8)$$

$$\frac{dy}{dx}=0---(18)$$

As the strain is (zero), these positions correspond to antinodes. $\sin\frac{2\pi x}{\lambda}=\pm 1$ **But** $\sin(2m+1)\frac{\pi}{2}=\pm 1$ **Where m=0, 1, 2, 3… etc.**

$$(2m+1)\frac{\pi}{2}=\frac{2\pi x}{\lambda} \quad or \quad x=\frac{(2m+1)\lambda}{4}$$

$$x=\frac{\lambda}{4},\frac{3\lambda}{4},\frac{5\lambda}{4}......etc.$$

Thus, the antinodes are equidistant and separated by λ/2. The first antinode is at a distance of λ/4 from the closed end of the closed end pipe or fixed end of a stretched string.

Changes with respect to time:
(i) Consider the time, where

$$\sin\frac{2\pi\upsilon t}{\lambda}=0 \quad and \quad \cos\frac{2\pi\upsilon t}{\lambda}=\pm 1$$

$$y=-2a\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi\upsilon t}{\lambda}---(4)$$

$$y=\pm 2a\sin\frac{2\pi x}{\lambda}$$

$$A=2a\cos\frac{2\pi x}{\lambda}---(5)$$

$$A=2a\cos\frac{2\pi x}{\lambda}$$

$$\frac{dy}{dt}=\frac{4\pi a\upsilon}{\lambda}\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi\upsilon t}{\lambda}---(6)$$

$$\frac{dy}{dt}=0$$

$$\frac{d^2y}{dt^2}=\frac{8\pi^2 a\upsilon^2}{\lambda^2}\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi\upsilon t}{\lambda}---(7)$$

$$\frac{d^2y}{dt^2}=\frac{\mp 8\pi^2 a\upsilon^2}{\lambda^2}\sin\frac{2\pi x}{\lambda}$$

$$\frac{dy}{dx}=\frac{-4\pi a}{\lambda}\cos\frac{2\pi x}{\lambda}\cos\frac{2\pi\upsilon t}{\lambda}---(8)$$

$$\frac{dy}{dx}=\frac{\mp 4\pi a}{\lambda}\cos\frac{2\pi x}{\lambda}$$

$$\cos\frac{2\pi\upsilon t}{\lambda}=\pm 1$$

But $\cos(m\pi)=\pm 1$ **Where m=0, 1, 2, 3… etc.**

But, $\frac{\upsilon}{\lambda}=n=\frac{1}{T}$

$$\frac{2\pi\upsilon t}{\lambda}=m\pi \quad or \quad t=\frac{m\lambda}{2\upsilon}$$

$$t=\frac{mT}{2} \quad or \quad t=0,\frac{T}{2},T,\frac{3T}{2}...etc.$$

At these instants, each particle is at its extreme position and the pattern is stationary at that instant. This instant is called the stationary instant.

Changes with respect to time:
(i) Consider the time, where

$$\sin\frac{2\pi\upsilon t}{\lambda}=\pm 1 \quad and \quad \cos\frac{2\pi\upsilon t}{\lambda}=0$$

$$y=-2\,a\,\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi\upsilon t}{\lambda}---(4)$$

$$y=0$$

$$A=2\,a\,\cos\frac{2\pi x}{\lambda}---(5)$$

$$A=2\,a\,\cos\frac{2\pi x}{\lambda}$$

(independent of time)

$$\frac{dy}{dt}=\frac{4\pi\,a\upsilon}{\lambda}\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi\upsilon t}{\lambda}---(6)$$

$$\frac{dy}{dt}=\frac{\pm 4\pi\,a\upsilon}{\lambda}\cos\frac{2\pi x}{\lambda}$$

$$\frac{d^2y}{dt^2}=\frac{8\pi^2a\upsilon^2}{\lambda^2}\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi\upsilon t}{\lambda}---(7)$$

$$\frac{d^2y}{dt^2}=0$$

$$\frac{dy}{dx}=\frac{-4\pi a}{\lambda}\cos\frac{2\pi x}{\lambda}\cos\frac{2\pi\upsilon t}{\lambda}---(8)$$

$$\frac{dy}{dx}=0$$

$$\sin\frac{2\pi\upsilon t}{\lambda}=\pm 1 \quad and \quad \cos\frac{2\pi\upsilon t}{\lambda}=0$$

But $\sin(2m+1)\frac{\pi}{2}=\pm 1$ **Where m=0, 1, 2, 3… etc.**

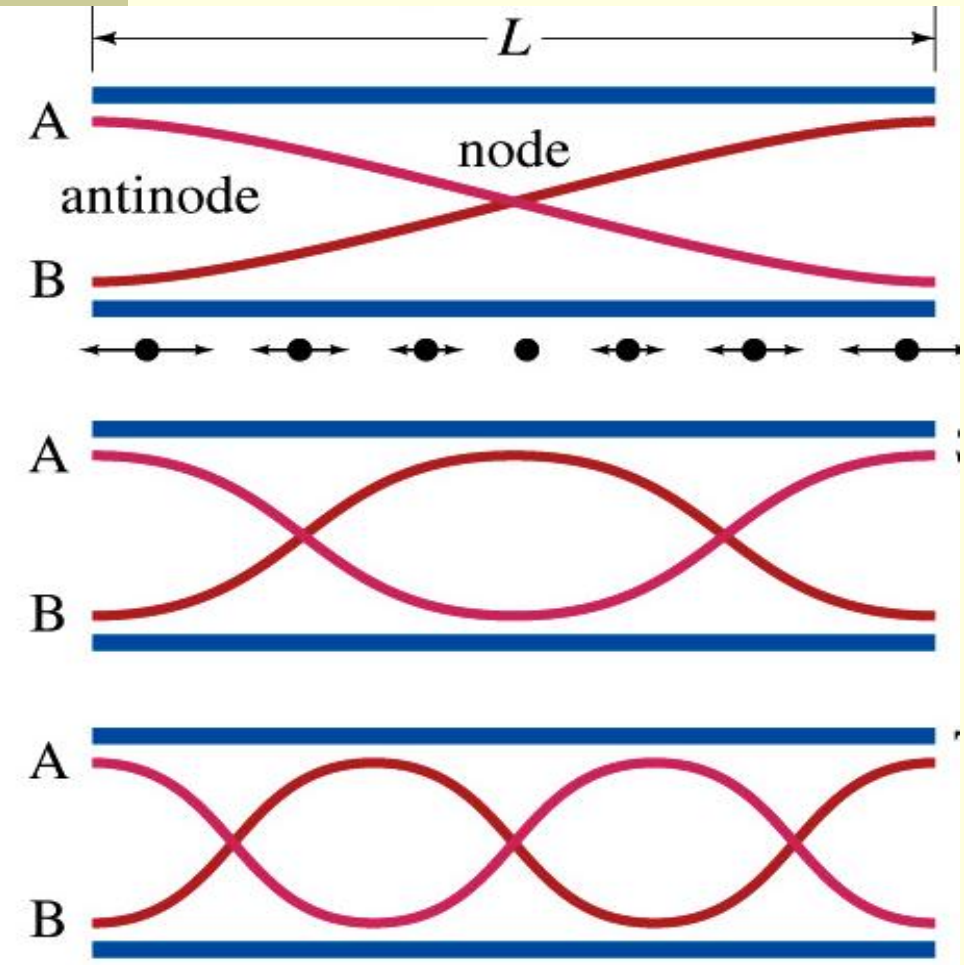
But, $\frac{\upsilon}{\lambda}=n=\frac{1}{T}$

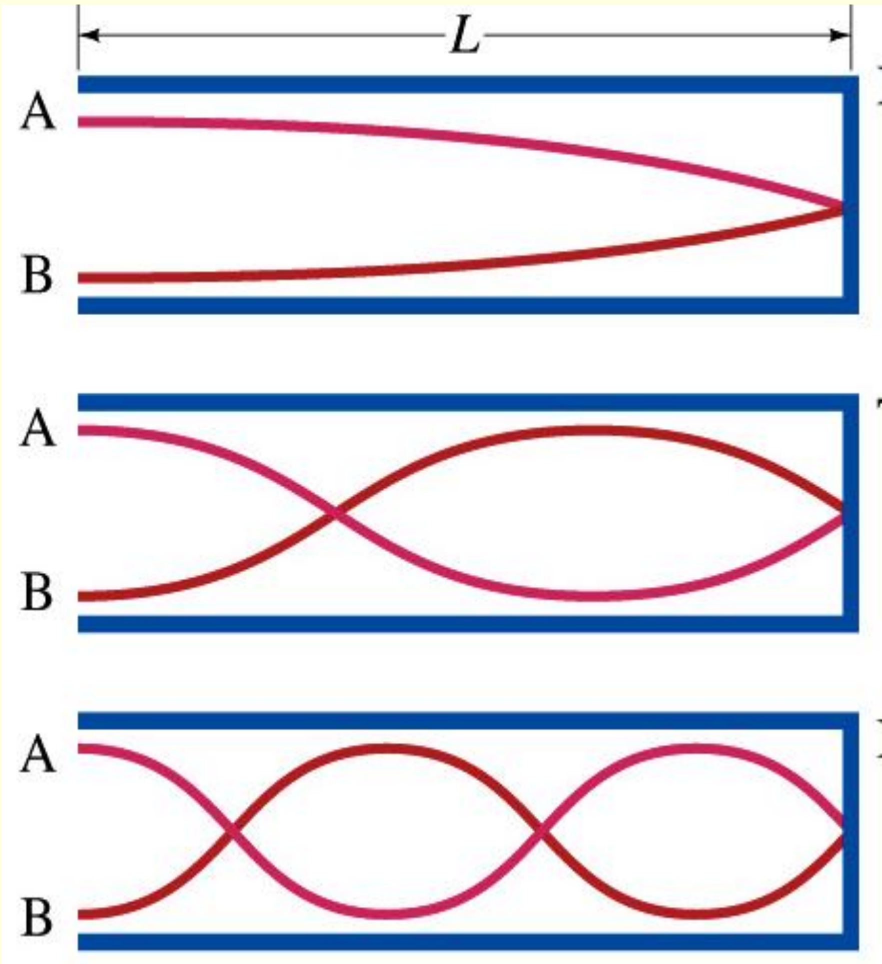
$$\frac{2\pi\upsilon t}{\lambda}=(2m+1)\frac{\pi}{2} \quad or \quad t=\frac{(2m+1)\lambda}{4\upsilon}$$

$$t=\frac{(2m+1)T}{2} \quad or \quad t=0,\frac{T}{4},\frac{3T}{4},\frac{5T}{4}\,...etc.$$

At these instants of time, all the particles are at their mean positions. It follows that twice in each time period the particles pass through their mean positions.

Waves in an Open-Open Pipe **Waves in an Open-Closed Pipe**





7.4: Energy of a Stationary Wave:

When a wave is propagated through a fluid;

$$E = \frac{-p}{dy/dx}$$

$$p = -E\,\frac{dy}{dx} - - - - (1)$$

$$\frac{dy}{dx} = \frac{-4\pi\ a}{\lambda}\sin\frac{2\pi x}{\lambda}\sin\frac{2\pi\upsilon t}{\lambda}$$

Also $\upsilon = \sqrt{\frac{E}{\rho}}$ $\qquad E = \upsilon^2\rho$

$$p = \upsilon^2\rho\,\frac{4\pi\ a}{\lambda}\sin\frac{2\pi x}{\lambda}\sin\frac{2\pi\upsilon t}{\lambda} - - - - - (2)$$

When $\sin\frac{2\pi x}{\lambda} = 1$ *and* $\sin\frac{2\pi\upsilon t}{\lambda} = 1$

$$p = p_0 = \upsilon^2\rho\,\frac{4\pi\ a}{\lambda} \quad - - - - - - (3)$$

$$p = p_0\sin\frac{2\pi x}{\lambda}\sin\frac{2\pi\upsilon t}{\lambda} - - - - - (4)$$

$$p_0\sin\frac{2\pi x}{\lambda} = p_x$$

$$p = p_x\sin\frac{2\pi\upsilon t}{\lambda} \quad - - - - - (5)$$

$$U = \frac{dy}{dt} = \frac{4\pi\ a\upsilon}{\lambda}\cos\frac{2\pi x}{\lambda}\cos\frac{2\pi\upsilon t}{\lambda} - - - - (6)$$

$$\frac{4\pi\ a\upsilon}{\lambda}\cos\frac{2\pi x}{\lambda} = U_x$$

$$U = U_x\ \cos\frac{2\pi\upsilon t}{\lambda} - - - - (7)$$

Work done or the energy transfer per unit area in a small interval of time ***dt = p U. dt***

The total energy transfer in time T = $\int_0^T p\ U\ dt$

Rate of energy transfer = $$\frac{\int_0^T p\,U\,dt}{T} = \frac{1}{T}\int_0^T \left(p_x \sin\frac{2\pi a\pi}{\lambda}\right)\left(U_x \cos\frac{2\pi a\pi}{\lambda}\right)dt$$

$$= \frac{p_x U_x}{T}\int_0^T \sin\frac{2\pi a\pi}{\lambda}\cos\frac{2\pi a\pi}{\lambda}\,dt$$

$$= \frac{p_x U_x}{2T}\int_0^T \sin\frac{4\pi a\pi}{\lambda}\,dt$$

$\sin 2\theta = 2\sin\theta\cos\theta$

But $$\int_0^T \sin\frac{4\pi a\pi}{\lambda}\,dt = 0$$

∴ Rate of energy transfer = 0

Thus in the case of stationary waves no energy is transferred.

7.5: Interference of Sound Waves:

If two or more sound waves travel through the same medium, superposition of the waves takes place.

The resultant vibration of the particles will depend upon the amplitude, time period, phase difference and direction of the interfering waves.

Consider two simple harmonic waves of the same frequency. Let *a* and *b* be the amplitudes of the two waves and phase difference ϕ

$$y_1 = a\sin\frac{2\pi}{\lambda}(\upsilon t - x) ------(1)$$

$$y_2 = b\sin\left(\frac{2\pi}{\lambda}(\upsilon t - x) + \phi\right) ------(2)$$

$$y = a\sin\frac{2\pi}{\lambda}(\upsilon t - x) + b\sin\left(\frac{2\pi}{\lambda}(\upsilon t - x) + \phi\right)$$

$$y = a\sin\frac{2\pi}{\lambda}(\upsilon t - x) + b\sin\frac{2\pi}{\lambda}(\upsilon t - x)\cos\phi + b\cos\frac{2\pi}{\lambda}(\upsilon t - x)\sin\phi$$

$$y = \left(\sin\frac{2\pi}{\lambda}(\upsilon t - x)\right)(a + b\cos\phi) + \left(\cos\frac{2\pi}{\lambda}(\upsilon t - x)\right)(b\sin\phi)$$

a+b cos Φ = A cos ϴ
b sin Φ = A sin ϴ

$$\therefore \quad A = \sqrt{a^2 + b^2 + 2ab\cos\phi} \quad ------(3)$$

$$\tan\theta = \frac{b\sin\phi}{a + b\cos\phi} ------(4)$$

$$\therefore \quad y = \left(\sin\frac{2\pi}{\lambda}(\upsilon t - x)\right)(A\cos\theta) + \left(\cos\frac{2\pi}{\lambda}(\upsilon t - x)\right)(A\sin\theta)$$

$$y = A\sin\left(\frac{2\pi}{\lambda}(\upsilon t - x) + \theta\right) \quad ------(5)$$

$$y = A\sin\left(\frac{2\pi}{\lambda}(\upsilon t - x) + \theta\right) \quad ------(5)$$

The resultant vibration has the same time period, (i.e. frequency and wavelength). The amplitude and phase difference are different.

(1) When: Φ=0, 2π, n (2 π) ….etc. And a=b,
A= $(a^2+a^2+2a^2)^{1/2}$ Or A=2a
Intensity of sound, I=A^2=$4a^2$
Therefore the intensity of sound is a maximum at those points, where the two waves differ in phase by n (2 π) or the phase difference is nλ.
Here n=0, 1, 2, 3… Etc.

(2) When: Φ = π, 3 π, (2n+1) π ….etc. And a=b,
A= 0
Intensity of sound, I=A^2=0
Therefore the intensity of sound is a minimum at those points, where the two waves differ in phase by (2n+1) π or the phase difference is (2n+1) λ /2.
Here n=0, 1, 2, 3… Etc.

7.6: Special Cases:

(1) When two waves of the same frequency and amplitude travel in the same direction having phase difference (zero):

Here a=b and Φλ=0
From equation (3);
A=2a
From equation (4);
tan ϴ=0 or ϴ =0
From equation (5);

$$y = 2a\sin\frac{2\pi}{\lambda}(\upsilon t - x) \quad - - - - - (6)$$

The resultant vibration has amplitude 2a as shown in figure (7).
In this conditions and rarefactions are coincident.

(2) When two waves of the same frequency and amplitude travel in the same direction having phase difference (π):

Here a=b and Φ =π
From equation (3);
A=0
From equation (4);
tan ϴ=0 or ϴ =0
From equation (5);
y=0

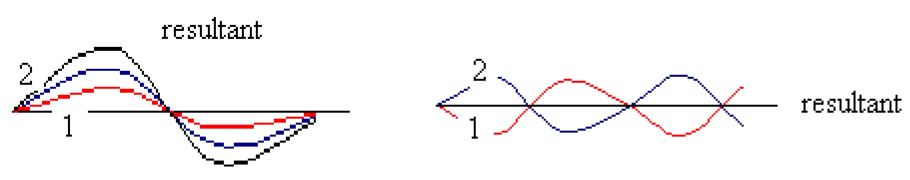


Figure (7) Figure (8)

The resultant vibration has zero amplitude and medium remains undisturbed as shown in figure (8).

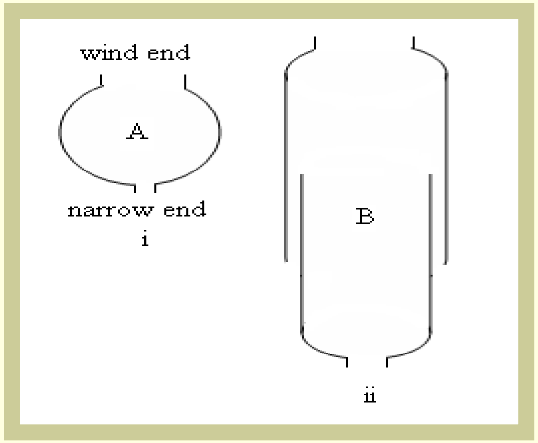
(3) When two waves of the same frequency and amplitude travel in opposite directions, the interference phenomenon gives rise to standing or stationary waves as discussed earlier.

7.7: Conditions for Interference of Sound Waves:

- 1-The two sources of sound must be coherent. They emit waves of the same frequency and amplitude.
- 2-The phase difference between the two sources must remain constant such that the phase difference between the two waves at any point does not change with time.
- 3-The two wave trains must travel in the same direction.
- Condition (1) is necessary so that the positions of maximum and minimum intensity of sound are distinct. If the difference between a and b is large, the minimum intensity positions are not distinct.
- Condition (2) is necessary to have sustained maxima and minima. If the phase difference between the two waves continuously changes, the maximum and minimum intensity positions are not fixed.
- Condition (3) is necessary because with increase in obliquity between the waves, the resultant intensity decreases.

7.8: Helmholtz Resonator:

- In various acoustical investigations, Helmholtz resonators are commonly used. Helmholtz particularly used them in the study of the equality of musical notes and hence these are called helmholtz resonators.
- The principle of a resonator is that the air contained in a hollow body with a narrow neck, say a bottle, and vibrates with a certain time period. If a tuning fork of the same frequency as that of the natural frequency of a resonator is held near its mouth, the sound intensity increases and resonator vibrations are heard. Two common forms Helmholtz resonators are shown in the following figure. Both the resonators are provided with a wide and a narrow mouth. The source of sound is kept at the wide end, and the ear is kept near the narrow end.
- The natural frequency of resonator A is fixed because of its fixed volume of air. In the case of B, the volume of air enclosed can be varied as desired.



7-9Theory of Resonator:

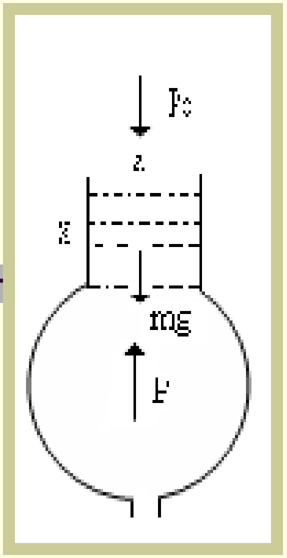
Consider a flask containing air and narrow cross section area (a).

Let the volume of the flask (V) and the mass of air contained in the neck be (m).

The pressure inside the flask is (P) and outside pressure is (P_0)

$$P = P_0 + \frac{mg}{a} - - - (1)$$

لقد افترض العالم الانكليزى اللورد رالى : فى حالة كون الابعاد للعنق المرنان صغيرة بالمقارنة مع الطول الموجى للصوت الذى يستجيب له المرنان فان الهواء فى عنق المرنان يتحرك كقطعة واحدة اى كالسداد او المكبس تماما. واذا كانت هذة الحركة سريعة كامهتز سريع فان العملية تكون كضمبة(اديباتيكية).



Let, any instant, the mass of air in the neck move down through a distance **x**. If the compression is adiabatic, the new pressure P_1 in the vessel will be,

$$P_1 (V - ax)^{\gamma} = PV^{\gamma} - - - - (2)$$

$$\therefore P_1 = P\left(\frac{V}{V - ax}\right)^{\gamma}$$

$$P_1 = P\left(1 + \frac{a\,x}{V - ax}\right)^{\gamma}$$

$$or \quad P_1 = P\left(1 + \frac{\gamma\, a\, x}{V - ax}\right) \quad (approximation)$$

$$\therefore \quad P_1 - P = \frac{p\,\gamma\, a\, x}{V - ax} \quad - - - - - - - (3)$$

The net downward force F on the air in the resonator;

$$F = \left(\left(P_0 + \frac{mg}{x}\right) - P_1\right) a$$

$$F = (P - P_1) a$$

$$F = -\frac{p\,\gamma\, a^2 x}{V - ax}$$

As ax is extremely small as compared to V

$$\therefore \; F = -\frac{p\,\gamma\, a^2 x}{V}$$

$$\therefore \; F = -\frac{p\,\gamma\, a^2 x}{V} = \mathrm{Mass} \times acceleration$$

$$\therefore \; acceleration = -\frac{p\,\gamma\, a^2}{mV} x$$

$$\therefore \frac{acceleration}{displacement} = -\frac{p\,\gamma\, a^2}{mV} \quad - - - - (4)$$

This represents simple harmonic motion.

$$\therefore \frac{acceleration}{displacement} = -\frac{p\gamma a^2}{mV} \quad ----(4)$$

This represents simple harmonic motion.

$$T = 2\pi\sqrt{\frac{displacement}{acceleration}} = \qquad T = 2\pi\sqrt{\frac{mV}{p\gamma a^2}}$$

The frequency of oscillation;

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{p\gamma a^2}{mV}} \quad -----(5)$$

The velocity of sound in air;

$$\upsilon = \sqrt{\frac{p\gamma}{\rho}} \quad or \quad P\gamma = \upsilon^2\rho$$

$$f = \frac{1}{2\pi}\sqrt{\frac{\upsilon^2\rho a^2}{mV}}$$

But, mass of air, m=alρ Here l is the length of the neck.

$$f = \frac{1}{2\pi}\sqrt{\frac{\upsilon^2\rho a^2}{al\rho V}} \qquad f = \frac{\upsilon}{2\pi}\sqrt{\frac{a}{lV}} \quad ---(6)$$

$$f^2 V = \frac{\upsilon^2 a}{4\pi^2 l} \quad ---(7)$$

As a, l and ρ remain constant, f^2V=constant

$$\therefore \quad f \propto \frac{1}{\sqrt{V}}$$

Thus, the frequency of the resonator is inversely proportional to the square root of its volume.

7-10 Dependence of the Frequency of Resonator on the Size and Shape of the Mouth:

The frequency of a Holmholtz resonator also depends upon the size and shape of the mouth. The time taken by the excess of pressure inside to drive out air depends upon the size of mouth of the resonator. If air can escape more easily (winder mouth) the time period is short i.e., the frequency is high and vice versa. The property of the mouth to allow the air to escape less easily or more easily is denoted by the term acoustic conductivity of the mouth. This term acoustic conductivity depends upon the area of cross section (a) and the length (l) of the mouth.

The acoustic conductivity;

$$k = \frac{a}{l} - - - - - (1)$$

$$f = \frac{\upsilon}{2\pi}\sqrt{\frac{a}{lV}} = \frac{\upsilon}{2\pi}\sqrt{\frac{k}{V}} - - - - (2)$$

For a circular aperture in a thin wall, l is very small. Raleigh showed that in such cases, the acoustic conductivity k is approximately equal to the diameter.

$$k = \frac{a}{l} = 2r$$

$$f = \frac{\upsilon}{2\pi}\sqrt{\frac{2r}{V}} - - - - (3)$$

7-11 Intensity of Sound Waves

The intensity of sound is defined as the average rate of transfer of energy per unit area. Determination of the intensity of sound is important in practical acoustics.

Amount of energy transfer per unit area per second $= 2\pi^2 \rho n^2 a^2 v ----(1)$

$$\upsilon = \sqrt{\frac{E}{\rho}}$$

$$E = \upsilon^2 \rho -----------(2)$$

$$E = \frac{-p}{dy/dx}$$

$$p = -E\frac{dy}{dx} -----------(3)$$

$$y = a\sin\frac{2\pi}{\lambda}(vt - x)$$

$$\frac{dy}{dx} = \frac{-2\pi\, a}{\lambda}\cos\frac{2\pi}{\lambda}(vt-x) --------(4)$$

$$p = \upsilon^2 \rho \frac{2\pi\, a}{\lambda}\cos\frac{2\pi}{\lambda}(vt-x) ----------(5)$$

$$p = p_{max.} = \upsilon^2 \rho \frac{2\pi\, a}{\lambda} ------(6)$$

$$p = p_{max.}\cos\frac{2\pi}{\lambda}(vt-x) ----------(7)$$

$$p_{max.} = 2\pi a \rho v n -------------(8)$$

$$I = \frac{(2\pi a \rho v n)^2}{2\rho v}$$

$$I = \frac{(p_{max.})^2}{2\rho v}$$

$$I = \frac{\Delta E}{A\, \Delta t} = \frac{P}{A}$$

Measurement of Intensity Level

The intensity level or the decibel level of the sound, *b*:
The *bel* is a measure of the ratio of energies.