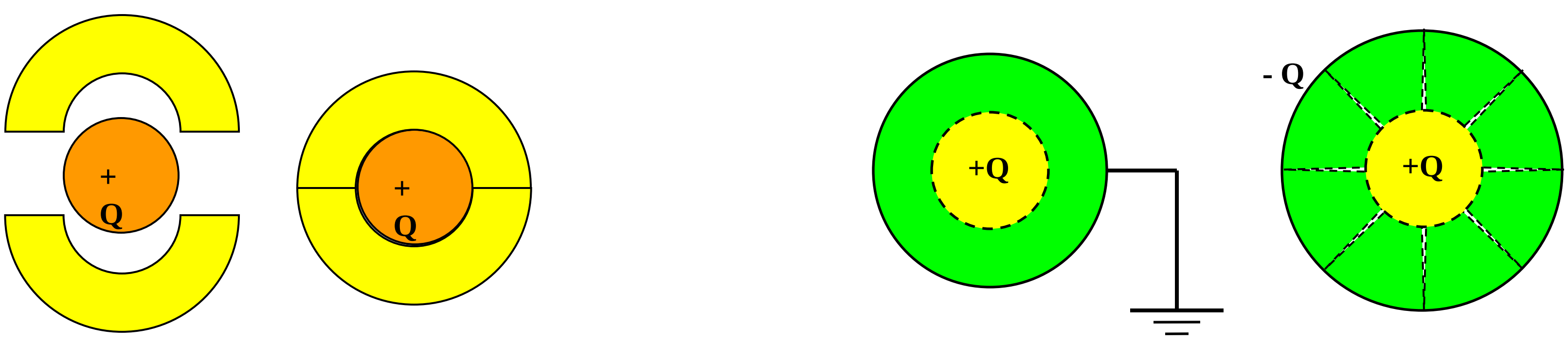
3-5: Electric Flux Density (Electric Displacement): **To demonstrate the concept of the electric flux density, the following simple experimental steps are required:**

(1). Suppose apply an amount of charge (+Q) to a metallic sphere of radius (a),

(2). enclose this charged sphere using a pair of connecting hemi-spheres with radius (b), (b>a), being very careful not to ever let any part of the outer sphere come in contact with the inner sphere,

(3). briefly ground the outer sphere,

(4). Then remove the ground connection and finding that (- Q) of charges has accumulated on the outer sphere. Somehow, the (+Q) charge of the inner sphere has induced the (- Q) charge on the outer sphere.



a b c d

Then we can say that the electric flux (Ψ) in Coulombs begins at the (+Q) charge and terminate at the (-Q) charge. These lines will be radially directed away from the inner sphere to the outer sphere and will be spread themselves out to get maximum separation between the like charges on each sphere.

Considering that the flux lines pass through a spherical surface $4\pi r^2$ in the region between the sphere, we can define the electric flux density $\vec{\mathbf{D}}$ $in\ (C/m^2)$ as:

$$\vec{\mathbf{D}} = \frac{\psi}{4\pi r^2}\hat{a}_r = \frac{Q}{4\pi r^2}\hat{a}_r$$

$$\vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_\circ r^2}\hat{a}_r$$

$$\vec{\mathbf{D}} = \varepsilon_\circ \vec{\mathbf{E}} \quad in\ free\ space$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \quad in\ material\ medium$$

$$\varepsilon = \varepsilon_\circ \varepsilon_r$$

$\varepsilon_\circ = permittivity\ of\ free\ space$

$\varepsilon_r = relative\ permittivity$

$\varepsilon = dielectric\ constant\ of\ the\ medium$

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_\circ}$$

(Ψ), is the number of electric flux lines and it equals to the amount of (+ Q) charges emanating electric lines, then:

$$\psi = Q\ (in\ Coulomb)$$

These equations indicate that the electric field intensity and electric flux density are related to each other through the permittivity of the medium and are pointed to the same direction.

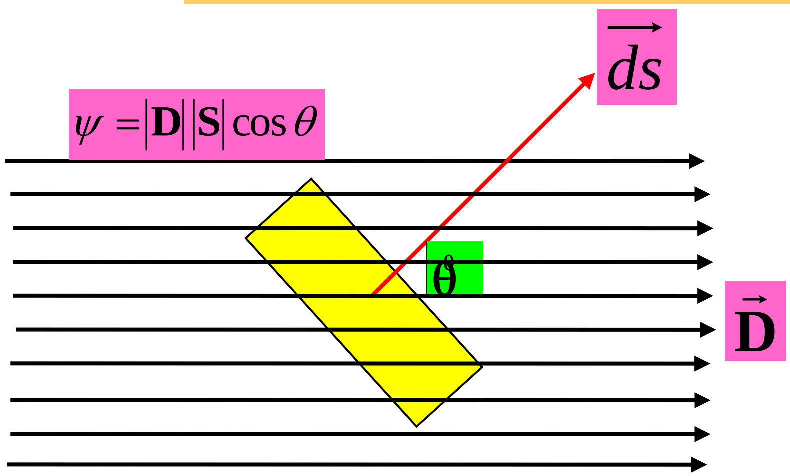
However, the amount of flux passing through a surface is given by the product of $\vec{\mathbf{D}}$ **and the amount of surface** $\overrightarrow{\mathbf{ds}}$ *normal to* $\vec{\mathbf{D}}$ **According to the figure shown below, we can see that the flux is given by:**

$$\vec{\mathbf{D}} = \frac{\psi}{4\pi r^2}\hat{a}_r = \frac{\psi}{S}\hat{a}_r \Rightarrow \psi = \vec{\mathbf{D}}\cdot\vec{\mathbf{S}} = |\mathbf{D}|\,|\mathbf{S}|\cos\theta \text{ -------(*)}$$

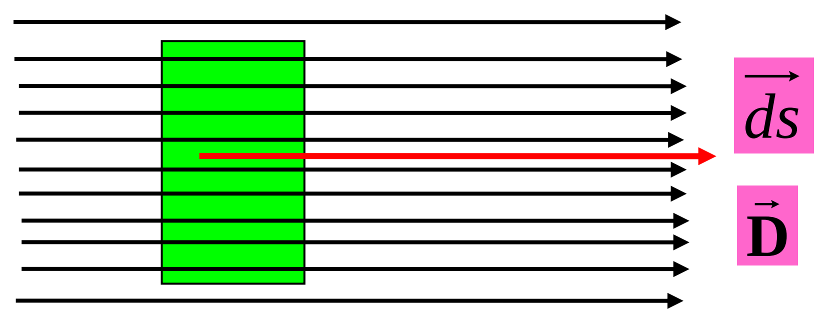
In addition, for arbitrary surface shapes, this equation can be more generalized to integration form as:

$$\psi = \oint_S \vec{\mathbf{D}}\cdot\overrightarrow{\mathbf{ds}} = Q_{enc} \qquad \overrightarrow{\mathbf{ds}} = ds\,\hat{a}_n$$

is also called the Gauss's Law



(ψ) is maximum , θ=0



3-6: Gauss's Law and Derivation of 1st Maxwell's Equation:

If we completely enclose a charge, then the net flux passing through the enclosing surface must be equal to the charge enclosed ($Q_{enc.}$). A formal statement of it is as follows:

" The net electric flux passing through any closed surface is equal to the total charge enclosed by that surface and it mathematically written as":

$$\oint_S \vec{\mathbf{D}} \cdot \overrightarrow{\mathbf{ds}} = Q_{enc}$$

Gauss's law is very simple compare to coulomb's law in finding electric field intensity for a given charge distributions of high degree of symmetry.

$$Q = \int_v \rho_v \, dv \; -------(1) \qquad and \qquad Q = \psi = \oint_s \vec{\mathbf{D}} \cdot \overrightarrow{\mathbf{ds}} \; --------(2)$$

$$hence: \int_v \rho_v \, dv = \oint_s \vec{\mathbf{D}} \cdot \overrightarrow{\mathbf{ds}} \; -------------(3)$$

and according to divergence theorem :

$$\oint_s \vec{\mathbf{A}} \cdot \overrightarrow{\mathbf{ds}} = \int_v (\vec{\nabla} \cdot \vec{\mathbf{A}}) \, dv$$

$$hence: \int \rho_v \, dv = \oint_v \vec{\nabla} \cdot \vec{\mathbf{D}} \, dv \qquad \Rightarrow \qquad \vec{\nabla} \cdot \vec{\mathbf{D}} = \rho_v$$

$\vec{\nabla} \cdot \vec{\mathbf{D}} = \rho_v$ **: is called first Maxwell's equation in point form.**

$$\int_v \rho_v \, dv = \oint_s \vec{\mathbf{D}} \cdot \overrightarrow{\mathbf{ds}}$$

is called first Maxwell's equation in integral form.

This equation means that (or states that), “ The divergence of the electric flux density at a given point per unit volume is equal to the charge density at that point”.

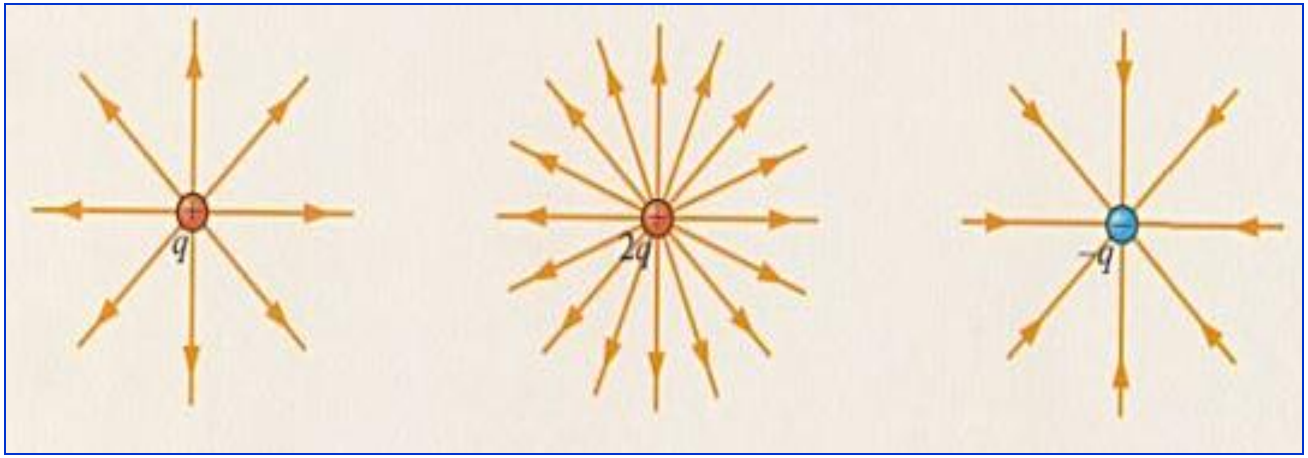
$$\vec{\nabla}\cdot\vec{\mathbf{D}} = \begin{cases} + & \text{existance of Source } (+Q) \\ 0 & \text{neither source nor sink} \\ - & \text{existance of sink } (-Q) \end{cases}$$

$$\vec{\nabla}\cdot\vec{\mathbf{D}} = \rho_v$$

$$\vec{\mathbf{D}} = \varepsilon\,\vec{\mathbf{E}}$$

$$\vec{\mathbf{E}}\,\frac{Q}{4\pi\,\varepsilon\,R^2}\,\hat{a}_R$$

$$Q = \int_v \rho_v\,dv$$



Gauss’s law is an alternative statement of Coulomb’s law, proper application of the divergence theorem to Coulomb’s law results in Gauss’s law.

To successfully apply Gauss’s law, the surface (S) should be chosen such that, from symmetry considerations, the magnitude of $\vec{\mathbf{D}}$ is constant and its direction is normal or tangential at every point of each surface of (S).

3-7: Gauss's Law and its Application:

Gauss's law provide an easy means of finding $\vec{\mathbf{E}}$ *and* $\vec{\mathbf{D}}$ for uniform symmetrical charge distributions such as: [point charge, infinite line charge, infinite sheet charge………].

3-7-1: Application on Point Charge:

Suppose a point charge (Q) is located at the origin. To determine $\vec{\mathbf{D}}$ *at a point P*, it is easy to see that choosing a spherical surface containing (P) will satisfy symmetry conditions. Thus, a spherical surface centered at the origin is the Gaussian surface in this case as shown below:

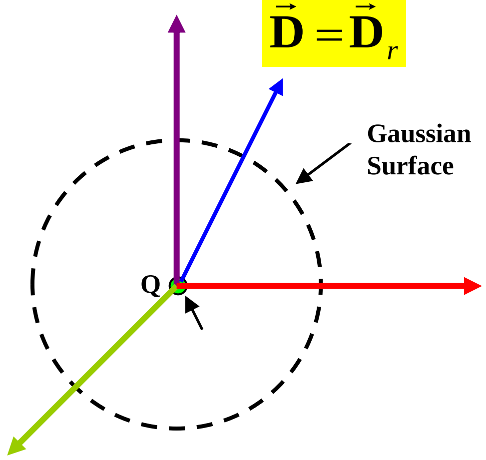
$$\vec{\mathbf{D}} = \mathbf{D}_r \, \hat{a}_r \qquad \overrightarrow{\mathbf{ds}} = r^2 \sin\theta \, d\theta \, d\phi \;\; \hat{a}_r$$

$$\oint_s \vec{\mathbf{D}} \cdot \overrightarrow{\mathbf{ds}} = Q_{enc} \qquad \Rightarrow \oint_s \mathbf{D}_r \, \hat{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi \;\; \hat{a}_r = Q_{enc}$$

$$\mathbf{D}_r \, (4\pi r^2) = Q_{enc} = Q$$

$$\mathbf{D}_r = \frac{Q}{4\pi r^2} \qquad \Rightarrow \therefore \vec{\mathbf{D}} = \frac{Q}{4\pi r^2} \hat{a}_r \quad and \quad \vec{\mathbf{E}} = \frac{Q}{4\pi \varepsilon r^2} \hat{a}_r$$

$$\vec{\mathbf{D}} = \vec{\mathbf{D}}_r$$



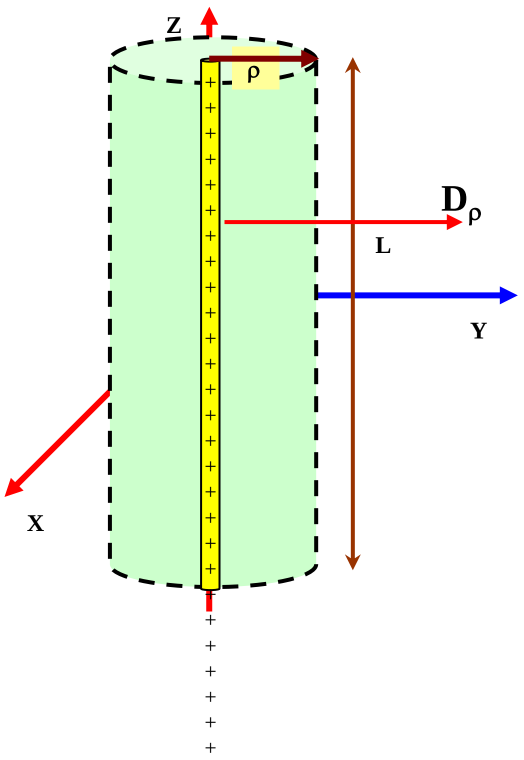
3-7-2: Application on Line Charge:

Suppose the infinite line of uniform charge $\rho_l \ (C/m)$ along the $z-axis$ lies. Determine electric flux density at point (P) as indicated in the figure below, by using Gauss's law.

$$\oint_S \vec{\mathbf{D}}\cdot\overrightarrow{\mathbf{ds}} = \oint_{top} \vec{\mathbf{D}}\cdot\overrightarrow{\mathbf{ds}} + \oint_{side} \vec{\mathbf{D}}\cdot\overrightarrow{\mathbf{ds}} + \oint_{bottom} \vec{\mathbf{D}}\cdot\overrightarrow{\mathbf{ds}}$$

$$\oint_{top} \vec{\mathbf{D}}\cdot\overrightarrow{\mathbf{ds}} = \int_0^{2\pi}\int_0^{\rho} \mathbf{D}_\rho \, \hat{a}_\rho \cdot \rho \, d\rho \, d\phi \ \hat{a}_z = zero$$

$$\oint_{bottom} \vec{\mathbf{D}}\cdot\overrightarrow{\mathbf{ds}} = \int_0^{2\pi}\int_0^{\rho} \mathbf{D}_\rho \, \hat{a}_\rho \cdot \rho \, d\rho \, d\phi \ (-\hat{a}_z) = zero$$



$$\oint_{side} \vec{\mathbf{D}}\cdot\overrightarrow{\mathbf{ds}} = \int_0^{L}\int_0^{2\pi} \mathbf{D}_\rho \hat{a}_\rho \cdot \rho \, dz \, d\phi \ \hat{a}_\rho = Q_{enc.}$$

$$\mathbf{D}_\rho (2\pi\rho L) = Q_{enc} \ --------- (1)$$

$$Q_{enc} = \int_L \rho_l \ dl = \rho_l \ L ------- (2)$$

$$Hence: \ \mathbf{D}_\rho (2\pi\rho L) = \rho_l \ L$$

Thus:

$$\vec{\mathbf{D}} = \frac{\rho_l}{2\pi\rho} \hat{a}_\rho \qquad and \qquad \vec{\mathbf{E}} = \frac{\vec{\mathbf{D}}}{\varepsilon} = \frac{\rho_l}{2\pi\varepsilon\rho} \hat{a}_\rho$$

3-7-3: Application on Infinite Plane Sheet Charge:

Consider the infinite sheet of uniform charge density $\rho_s\ (C/m^2)$ lying on the $z=0-plane$. Determine ($\vec{\mathbf{D}}$) at point (P) using Gauss's law.

We choose a rectangular box that is cut symmetrically by the sheet of charge and has two of its faces parallel to the sheet as shown in figure. As ($\vec{\mathbf{D}}=\mathbf{D}_z\,\hat{a}_z$) is normal to the sheet, and applying Gauss's law gives:

$$\oint_S \vec{\mathbf{D}}\cdot\overrightarrow{\mathbf{ds}} = \oint_{Top} \vec{\mathbf{D}}\cdot\overrightarrow{\mathbf{ds}} + \oint_{Bottom} \vec{\mathbf{D}}\cdot\overrightarrow{\mathbf{ds}} = Q_{enc} \qquad , \quad \overrightarrow{\mathbf{ds}} = dx\,dy\,\hat{a}_z$$

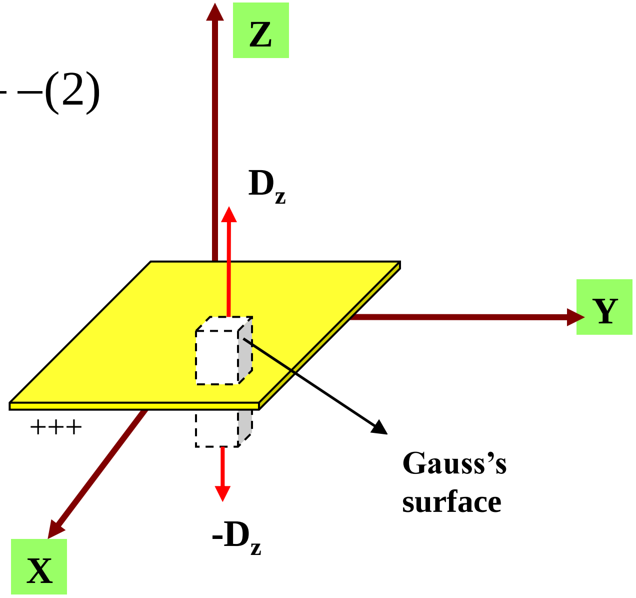
$$\oint_S \vec{\mathbf{D}}\cdot\overrightarrow{\mathbf{ds}} = \int_o^a\int_0^b \mathbf{D}_z\hat{a}_z\cdot dx\,dy\,\hat{a}_z + \int_o^a\int_0^b(-\mathbf{D}_z\hat{a}_z)\cdot dx\,dy\,(-\hat{a}_z) = \mathbf{D}_z\,(A+A) ------(1)$$

$$Q_{enc} = \int \rho_s\,ds = \rho_s\int dx\,dy = \rho_s\ A --------------------(2)$$

$$From\ \ eqs.(1)\ and\ \ (2)\ \ we\ \ get:$$

$$\mathbf{D}_z\,(2A) = \rho_s\ A \Rightarrow \quad \vec{\mathbf{D}} = \frac{\rho_s}{2}\hat{a}_z \quad and \quad hence \quad \vec{\mathbf{E}} = \frac{\rho_s}{2\varepsilon}\hat{a}_z$$

$$\vec{\mathbf{D}} = \frac{\rho_s}{2}\hat{a}_n \quad and \quad hence \quad \vec{\mathbf{E}} = \frac{\rho_s}{2\varepsilon}\hat{a}_n$$



3-7-4: Application on Spherical Charge:

Consider a sphere of radius (a) with a uniform volume charge density $\rho_v \ (C/m^3)$. Determine the electric flux density ($\vec{\mathbf{D}}$) everywhere using Gauss's law.

We construct Gaussian surface for cases $r \le a \quad and \quad r \ge a$ separately. Since the charge has spherical symmetry, it is obvious that a spherical surface is an appropriate Gaussian surface.

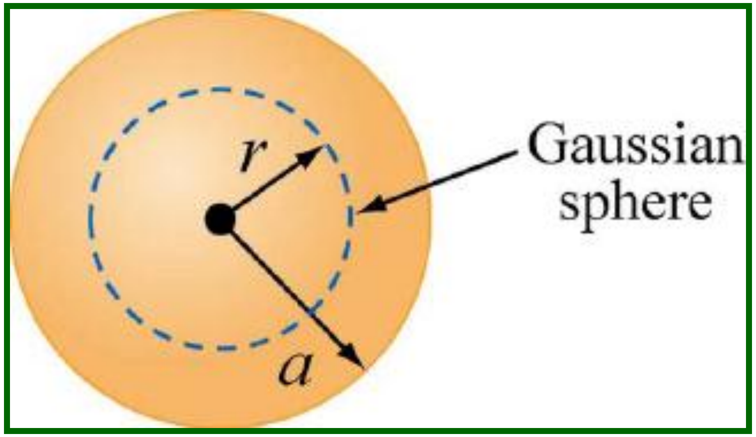
(1). For case $r \le a$, the total charge enclosed by the spherical surface of radius (r), is:

$$\oint_S \vec{\mathbf{D}} \cdot \overrightarrow{\mathbf{ds}} = Q_{enc} \quad , \quad \overrightarrow{\mathbf{ds}} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r \quad , \quad \vec{\mathbf{D}} = \mathbf{D}_r \, \hat{a}_r \quad , \quad Q_{enc} = \int \rho_v \, dv$$

$$\oint_S \vec{\mathbf{D}} \cdot \overrightarrow{\mathbf{ds}} = \int_0^{2\pi}\int_0^{\pi} \mathbf{D}_r \, \hat{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

$$\oint_S \vec{\mathbf{D}} \cdot \overrightarrow{\mathbf{ds}} = \mathbf{D}_r \, (4\pi r^2) \text{ ------------ } (1)$$

$$Q_{enc} = \int_0^{2\pi}\int_0^{\pi}\int_0^{r} \rho_v \, r^2 \sin\theta \, dr \, d\theta \, d\phi = \rho_v \frac{4\pi}{3} r^3 \text{ ------ } (2)$$



From eqs.(1) and (2) we get: $\vec{\mathbf{D}} = \dfrac{\rho_v \, r}{3} \, \hat{a}_r \quad and \quad hence \quad \vec{\mathbf{E}} = \dfrac{\rho_v \, r}{3\varepsilon} \, \hat{a}_r$

(2). For $r \geq a$, the Gaussian surface is shown in figure (b).

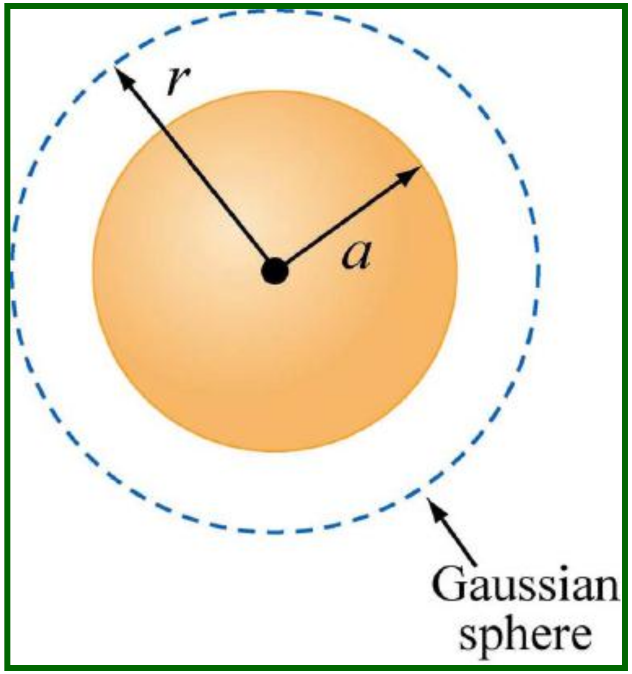
The charge enclosed by the surface is the entire charge and is given by:

$$\oint_S \vec{\mathbf{D}} \cdot \overrightarrow{\mathbf{ds}} = Q_{enc} \quad , \quad \overrightarrow{\mathbf{ds}} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r \quad , \quad \vec{\mathbf{D}} = \mathbf{D}_r \, \hat{a}_r \quad , \quad Q_{enc} = \int \rho_v \, dv$$

$$\oint_S \vec{\mathbf{D}} \cdot \overrightarrow{\mathbf{ds}} = \int_0^{2\pi} \int_0^{\pi} \mathbf{D}_r \, \hat{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

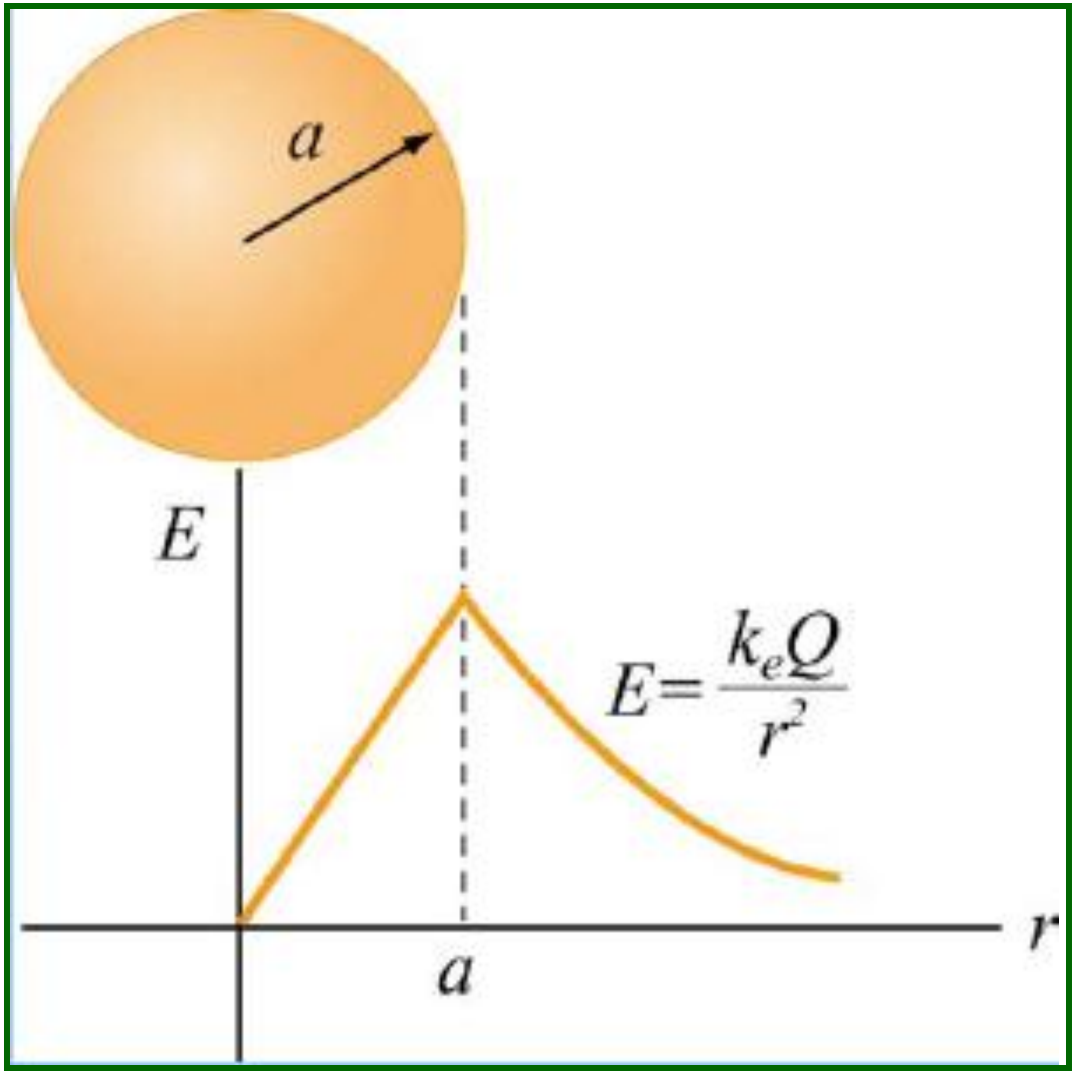
$$\oint_S \vec{\mathbf{D}} \cdot \overrightarrow{\mathbf{ds}} = \mathbf{D}_r \, (4\pi r^2) \text{ ------------(1)}$$

$$Q_{enc} = \int_0^{2\pi} \int_0^{\pi} \int_0^{a} \rho_v \, r^2 \sin\theta \, dr \, d\theta \, d\phi = \rho_v \frac{4\pi}{3} a^3 \text{ ------(2)}$$



From eqs.(1) and (2) we get: $\vec{\mathbf{D}} = \dfrac{\rho_v \, a^3}{3r^2} \hat{a}_r$ *and hence* $\vec{\mathbf{E}} = \dfrac{\rho_v \, a^3}{3\varepsilon r^2} \hat{a}_r$

The graphical representation of the value of the electric field intensity as a function of the Gaussian radius of a spherical charge is illustrated in this figure:



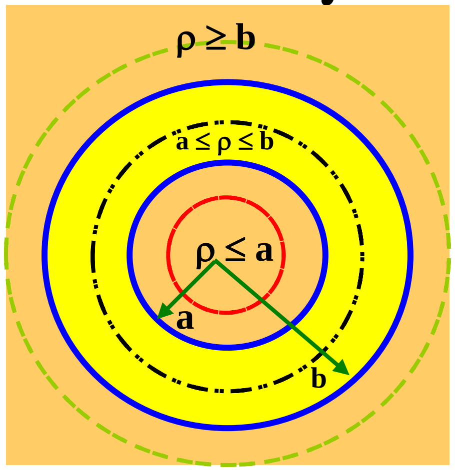
3-7-5: Application on Infinite Coaxial Cable:

Consider a coaxial cable of inner radius (a) and outer radius (b) as shown in figure. For practice in using Gauss's law, however, we will assume the charge on the inner conductor has a uniformly positive volume charge density of $\rho_v \ (C/m^3)$, and the outer conductor is grounded. Our task is to find the electric flux density everywhere.

To begin, we notice from the symmetry of the problem that $\vec{\mathbf{D}}$ only appears to be a function of (ρ) only:

(1). For the case at $\rho \le a$

$$\oint_S \vec{\mathbf{D}} \cdot \overrightarrow{\mathbf{ds}} = Q_{enc} \quad , \quad \overrightarrow{\mathbf{ds}} = \rho\, dz\, d\phi\, \hat{a}_\rho \quad , \quad \vec{\mathbf{D}} = \mathbf{D}_\rho\, \hat{a}_\rho \quad , \quad Q_{enc} = \int \rho_v\, dv$$



$$\oint_S \vec{\mathbf{D}} \cdot \overrightarrow{\mathbf{ds}} = \int_0^{2\pi}\int_0^{h} \mathbf{D}_\rho\, \hat{a}_\rho \cdot \rho\, dz\, d\phi\, \hat{a}_\rho$$

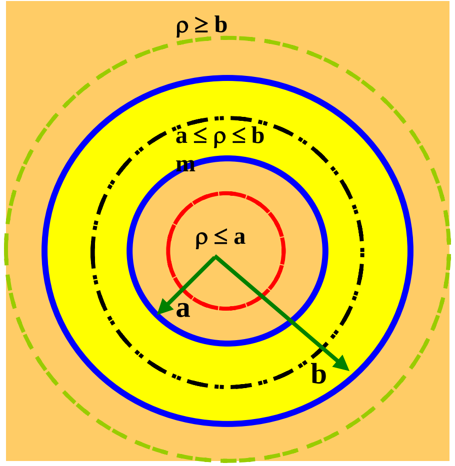
$$\oint_S \vec{\mathbf{D}} \cdot \overrightarrow{\mathbf{ds}} = \mathbf{D}_\rho\,(2\pi\rho h) -----------(1)$$

$$Q_{enc} = \int_0^{2\pi}\int_0^{h}\int_0^{\rho} \rho_v\ \rho\ d\rho\ dz\ d\phi = \rho_v\,(\pi\rho^2 h)\ -----(2)$$

From eqs.(1) and (2) we get: $\vec{\mathbf{D}} = \rho_v \dfrac{\rho}{2}\hat{a}_\rho \quad and \quad hence \quad \vec{\mathbf{E}} = \rho_v \dfrac{\rho}{2\varepsilon}\hat{a}_\rho$

(2). For the case at $a \leq \rho \leq b$

$$\oint_S \vec{\mathbf{D}} \cdot \overrightarrow{\mathbf{ds}} = Q_{enc} \quad , \quad \overrightarrow{\mathbf{ds}} = \rho\, dz\, d\phi\, \hat{a}_\rho \quad , \quad \vec{\mathbf{D}} = \mathbf{D}_\rho\, \hat{a}_\rho \quad , \quad Q_{enc} = \int \rho_v\, dv$$



$$\oint_S \vec{\mathbf{D}} \cdot \overrightarrow{\mathbf{ds}} = \int_0^{2\pi}\int_0^{h} \mathbf{D}_\rho\, \hat{a}_\rho \cdot \rho\, dz\, d\phi\, \hat{a}_\rho$$

$$\oint_S \vec{\mathbf{D}} \cdot \overrightarrow{\mathbf{ds}} = \mathbf{D}_\rho\,(2\pi\,\rho\, h) ------------(1)$$

$$Q_{enc} = \int_0^{2\pi}\int_0^{h}\int_0^{a} \rho_v\;\rho\; d\rho\; dz\; d\phi = \rho_v\,(\pi\, a^2\, h) ------(2)$$

From eqs.(1) and (2) we get: $\vec{\mathbf{D}} = \rho_v \dfrac{a^2}{2\rho}\, \hat{a}_\rho \quad and \quad hence \quad \vec{\mathbf{E}} = \rho_v \dfrac{a^2}{2\varepsilon\rho}\, \hat{a}_\rho$

(3). For the case at $\rho \geq b$

Since outer conductor is tied to ground, we know from Faraday's experiment that the charge on this conductor is (-Q). Then the total net charge enclosed by this third Gaussian surface is therefore zero (Q_{enc}= zero) and hence:

$$\vec{\mathbf{D}} = Zero$$

Example: **The volume in cylindrical coordinates between** $\rho = 2\,m \quad and \quad \rho = 4\,m$ **contains a uniform charge density** $\rho_v\,(C/m^3)$ **. Use Gauss's law to find** $\vec{D}$ **in all region ?**

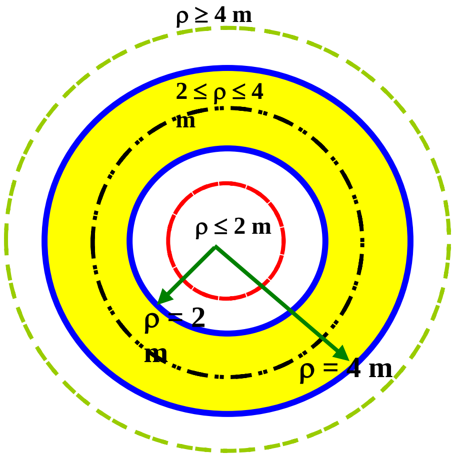
Solution

(1). For $0 \le \rho \le 2\,m$

$$\oint_S \vec{\mathbf{D}} \cdot \overrightarrow{\mathbf{ds}} = Q_{enc} = \int_v \rho_v \, dv ------------ Gauss's\ law$$

$$\oint_S \vec{\mathbf{D}} \cdot \overrightarrow{\mathbf{ds}} = \oint_S \mathbf{D}_\rho \, \hat{a}_\rho \cdot \rho \, d\phi \, dz \; \hat{a}_\rho = \mathbf{D}_\rho \int_0^{2\pi} \int_0^{L} \rho \, d\phi \, dz = \mathbf{D}_\rho (2\pi L \rho)$$

$$\oint_S \vec{\mathbf{D}} \cdot \overrightarrow{\mathbf{ds}} = \mathbf{D}_\rho (2\pi L \rho) --------- (1)$$



$$Q_{enc} = \int_v \rho_v \, dv = 0 ---------- (2)$$

Since there is no charges in this region
Equating eq.(1) with (2) we get:

$$\mathbf{D}_\rho (2\pi L \rho) = 0 \;\; \Rightarrow\Rightarrow \; \vec{\mathbf{D}} = 0 ----- (3)$$

(2). For $2 \le \rho \le 4m$

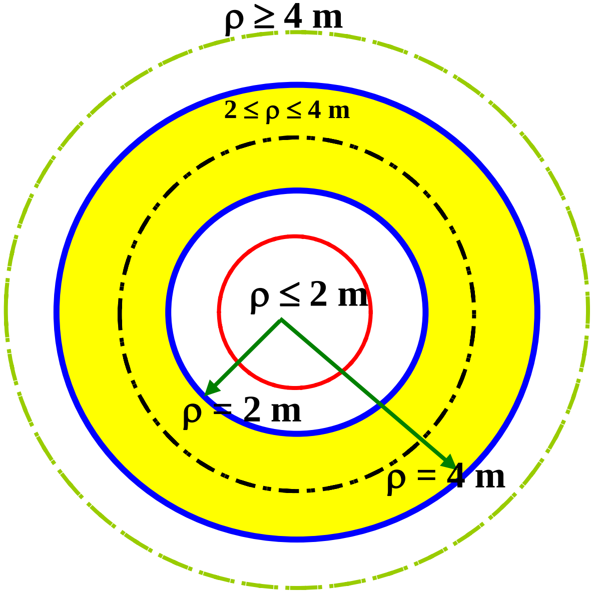
Equation (1) is remain the same as in the first case, while the total charge enclosed is calculated as:

$$Q_{enc} = \int_0^L \int_0^{2\pi} \int_2^{\rho} \rho_v \, \rho \, d\rho \, d\phi \, dz = \rho_v \left(\frac{\rho^2}{2} \right) \Big|_2^{\rho} (2\pi)\,(L) = \rho_v \, \pi \, L \, (\rho^2 - 4)$$

$$Q_{enc} = \rho_v \, \pi \, L \, (\rho^2 - 4) ----------(4)$$

Equating eq.(1) with (4) we get:

$$\mathbf{D}_{\rho} (2\pi L \rho) = (\rho_v \, \pi \, L \, (\rho^2 - 4) \Rightarrow\Rightarrow \vec{\mathbf{D}} = \frac{\rho_v \, (\rho^2 - 4)}{2\,\rho} \hat{a}_{\rho} ---(5)$$



(3). For $\rho \geq 4\, m$

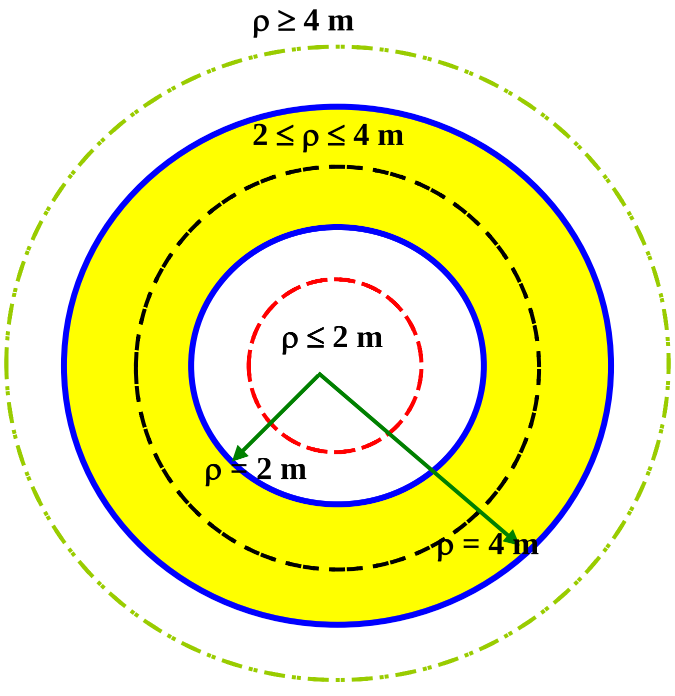
Equation (1) is remain the same as in the first and second cases, while the total charge enclosed is calculated as:

$$Q_{enc} = \int_0^L \int_0^{2\pi} \int_2^4 \rho_v \, \rho \, d\rho \, d\phi \, dz = \rho_v \left(\frac{\rho^2}{2} \right) \Big|_2^4 (2\pi)\, (L) = \rho_v \, \pi \, L \, (16-4)$$

$$Q_{enc} = 12 \, \rho_v \, \pi \, L - - - - - - - - - (6)$$

Equating eq.(1) with (5) we get:

$$\mathbf{D}_\rho (2\pi L \rho) = 12 \rho_v \pi L \;\Rightarrow\Rightarrow\; \vec{\mathbf{D}} = \frac{6\rho_v}{\rho} \, \hat{a}_\rho - - (7)$$



Home work

Q_1/ Charge is distributed in the spherical region $r \le 2\,m$ with density: $\rho_v = \frac{-200}{r^2}\ (\mu C/m^3)$. What net flux crosses the surfaces, $r = 1m \quad , \quad r = 4\ m \quad and \quad r = 500\ m$?

Q_2/ A point charge Q is at the origin of spherical coordinate and a sphere shell charge distribution at $r = a$ has a total charge of $(Q' - Q)$, uniformly distributed. What flux crosses the surface

$$r = k \quad for \quad k < a \quad and \quad k > a\,?$$

Q_3/ Given that: $\vec{D} = 2\rho\cos\phi\ \hat{a}_\phi - \frac{\sin\phi}{3\rho}\ \hat{a}_z\ \ (C/m^2)$, find the flux crossing the portion of the $z = 0$-plane defined by

$$0 \le \rho \le a \qquad , \qquad 0 \le \phi \le 90^\circ \qquad and \qquad 270^\circ \le \phi \le 360^\circ$$

.

Q_4/ A uniform line charge with density $\rho_l = 5\ (\mu C/m)$ lies along the x-axis. Find $\vec{D}$ at: $((3,2,1)?\quad Ans.\ [0.356(\frac{2\hat{a}_y + \hat{a}_z}{\sqrt{5}})$

.

Q_5/ Given charge distribution with density $\rho_v = 5\ r\ \ (C/m^3)$ in spherical coordinates. Use Gauss's law to find $\vec{D}$? Ans. $\vec{D} = \frac{5}{4} r^2\ \hat{a}_r\ \ (C/m^2)$

Q_6/ Given that: $\vec{D} = 500\, e^{-0.1x}\ \hat{a}_x\ \ (\mu C/m^2)$, find the flux ψ crossing surfaces of area $(1\ m^2)$ normal to the x-axis and located at $x = 1m\ ,\ x = 5\ m\ ,\ and\ \ x = 10\ m$? Ans. $(452\ ,\ 303\ ,\ 184)\,\mu C$

Q_7/ A point charge of Q is at the center of spherical coordinate system. Find the flux ψ which crosses an area of $4\pi\, m^2$ on a concentric spherical shell of radius $r = 3\ m?\ \ Ans.\ \ (\frac{Q}{9})$

Q_8/ An area of $40.2\ m^2$ on the surface of spherical shell of radius $r = 4\ m$ is crossed by $\psi = 10\ \mu C$ of flux in an inward direction. What point charge at the origin is indicated ? Ans. $(Q = -50\ \mu C)$